BSDF Compression Using Wavelets

Secret Weapons of RADIANCE: Stuff That Never Took Off

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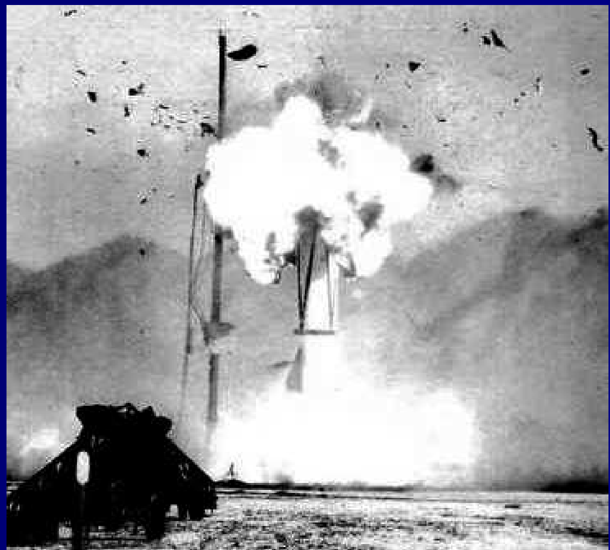
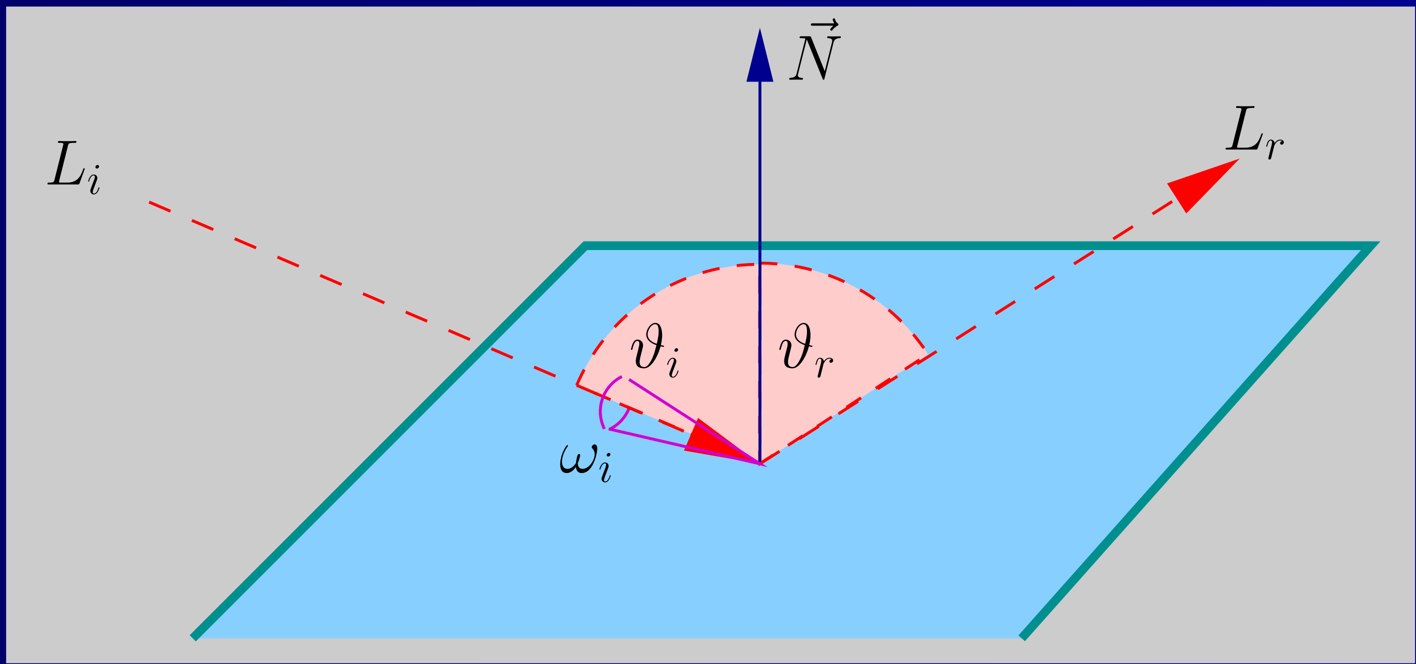


BSDF Compression Using Wavelets

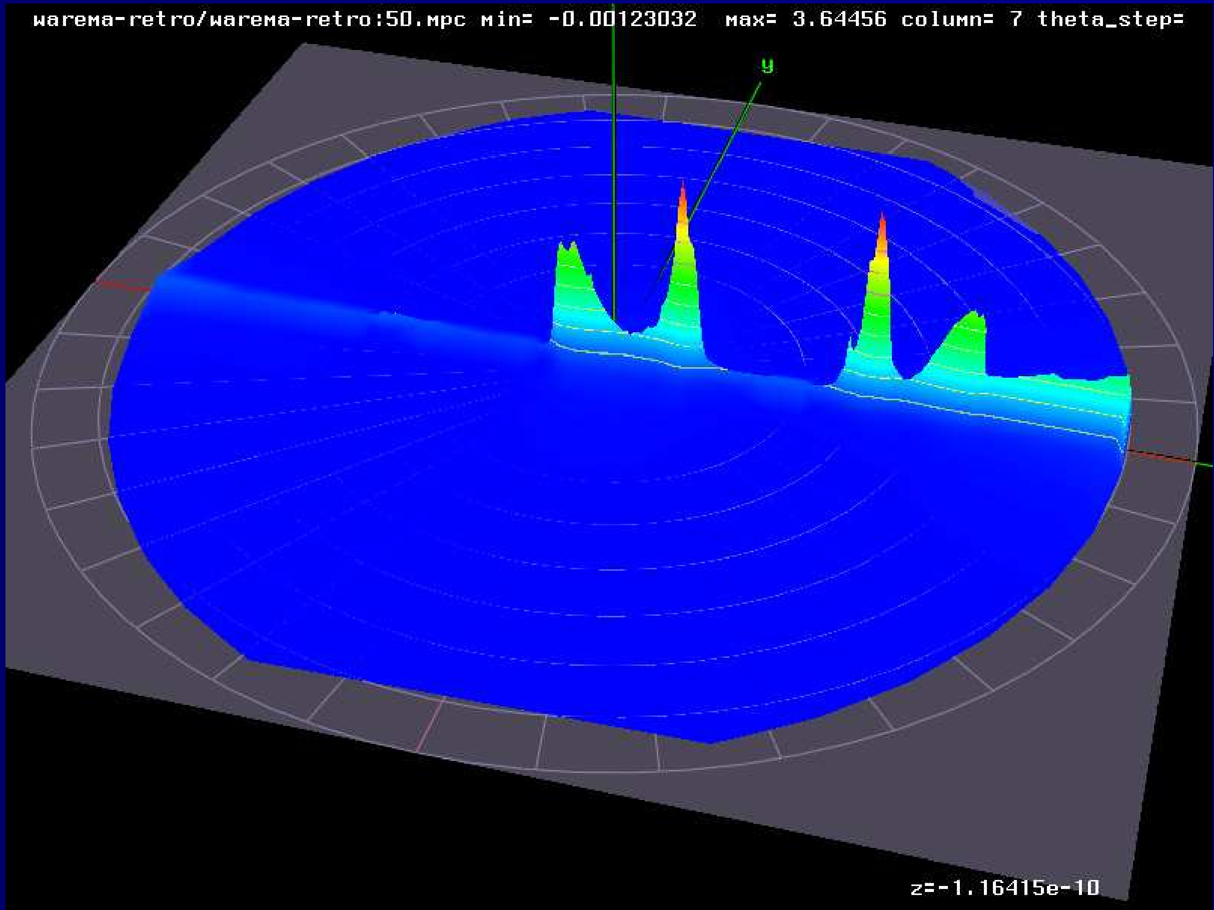
- Purpose: Efficiently store and retrieve measured 4D BSDF data for integration into RADIANCE through (lossy) wavelet compression
- Intended for photometric validation of RADIANCE / photon map
- Prototype developed winter 2000 – spring 2001
- Shelved summer 2001, abandoned & never published

BSDF

$$f\left(\vartheta_i, \varphi_i, \vartheta_r, \varphi_r\right) = \frac{L_r(\vartheta_r, \varphi_r)}{L_i(\vartheta_i, \varphi_i) \cos \vartheta_i \omega_i}$$

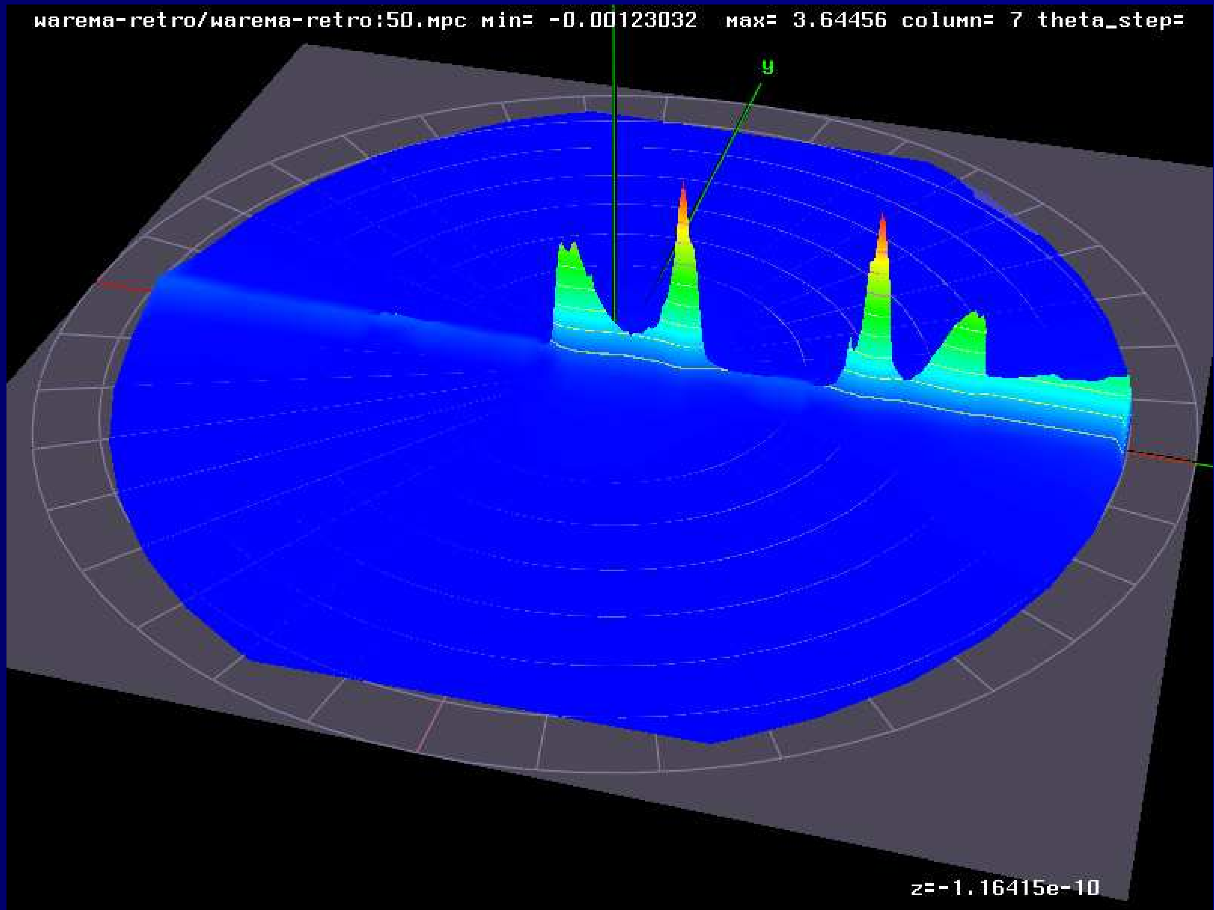


BSDF



RealLife™ BSDF: high angular resolution, large dataset

BSDF



$\Rightarrow$ Compression must preserve specular peaks!

Wavelet Theory

- Decompose signal into linear combinations of orthogonal 2D basis functions and coefficients
- Basis functions have finite support
  $\Rightarrow$ improved fidelity in peaks (compared to Fourier)
- Applications in:
  - Denoising, filtering (image & audio processing)
  - Trend analysis, statistics (stock exchange)
  - Computer Graphics (Wavelet Radiosity)
  - Compression (JPEG2000)

Wavelet Theory

Approximate signal $f\left(x\right)$ with 2^n samples by

$$\sum_{i=0}^{2^n-1} a_i\phi_i(x) + b_i\psi_i(x)$$

with

- Scaling func $\phi_i\left(\right)$ scaled by coeff a_i
- Wavelet func $\psi_i\left(x\right)$ scaled by coeff b_i
- ψ_i derived from linear combinations of ϕ_i
- ψ_i and ϕ_i *decorrelate* $f\left(x\right)$

Multiresolution Analysis

Represent $f(x)$ at varying detail levels $j \in \{0 \dots n\}$ by adapting ϕ_i, ψ_i in frequency domain:

$$f_j(x) = \sum_{i=0}^{2^j-1} a_{j,i}\phi_{j,i}(x) + b_{j,i}\psi_{j,i}(x)$$

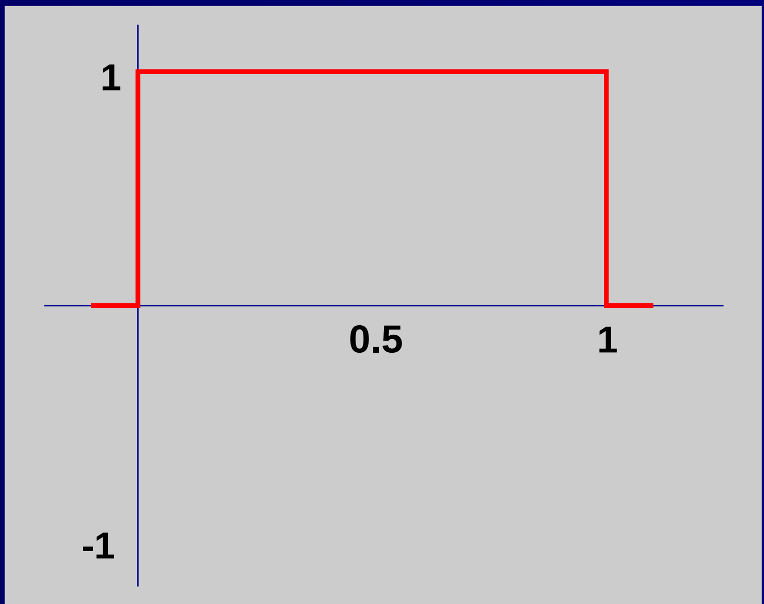
with

- $j = n$: $f_n(x) = f(x)$, original samples $s_0 \dots s_{2^n-1}$
- $j = 0$: Lowest resolution, single coefficient pair
- $j \to j-1$: Number of samples/coeffs halved
- $\phi_{j,i}$, $\psi_{j,i}$ dilated for res. j & translated for interval i

Basis Functions: Haar Wavelets

Scaling function dilated for resolution j and interval i:

$$\phi_{j,i}(x) = 2^j \left(x - i2^{-j}\right) \phi_{[0,1[} = \begin{cases} 1 & \text{if } \; i2^{-j} \leq x < (i+1)\, 2^{-j} \\ 0 & \text{else} \end{cases}$$

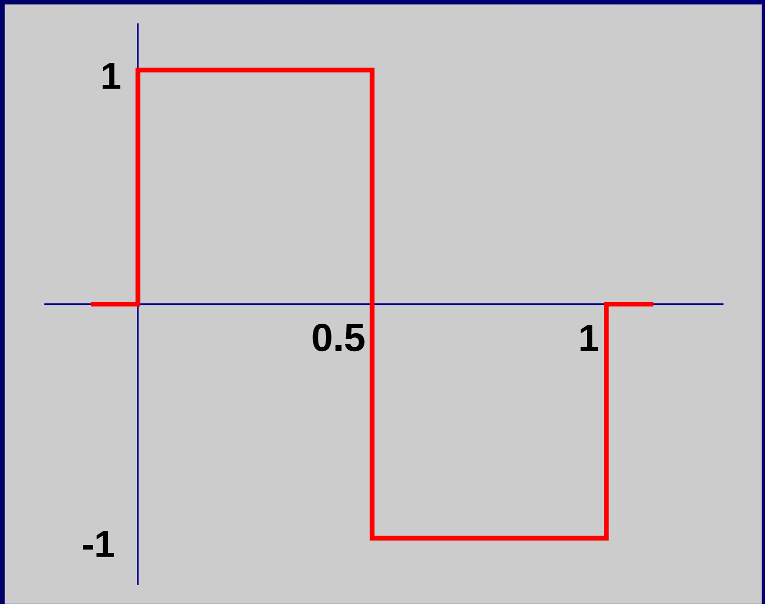


$\Rightarrow$ Average of neighbouring samples/coeffs at res. $j+1$

Basis Functions: Haar Wavelets

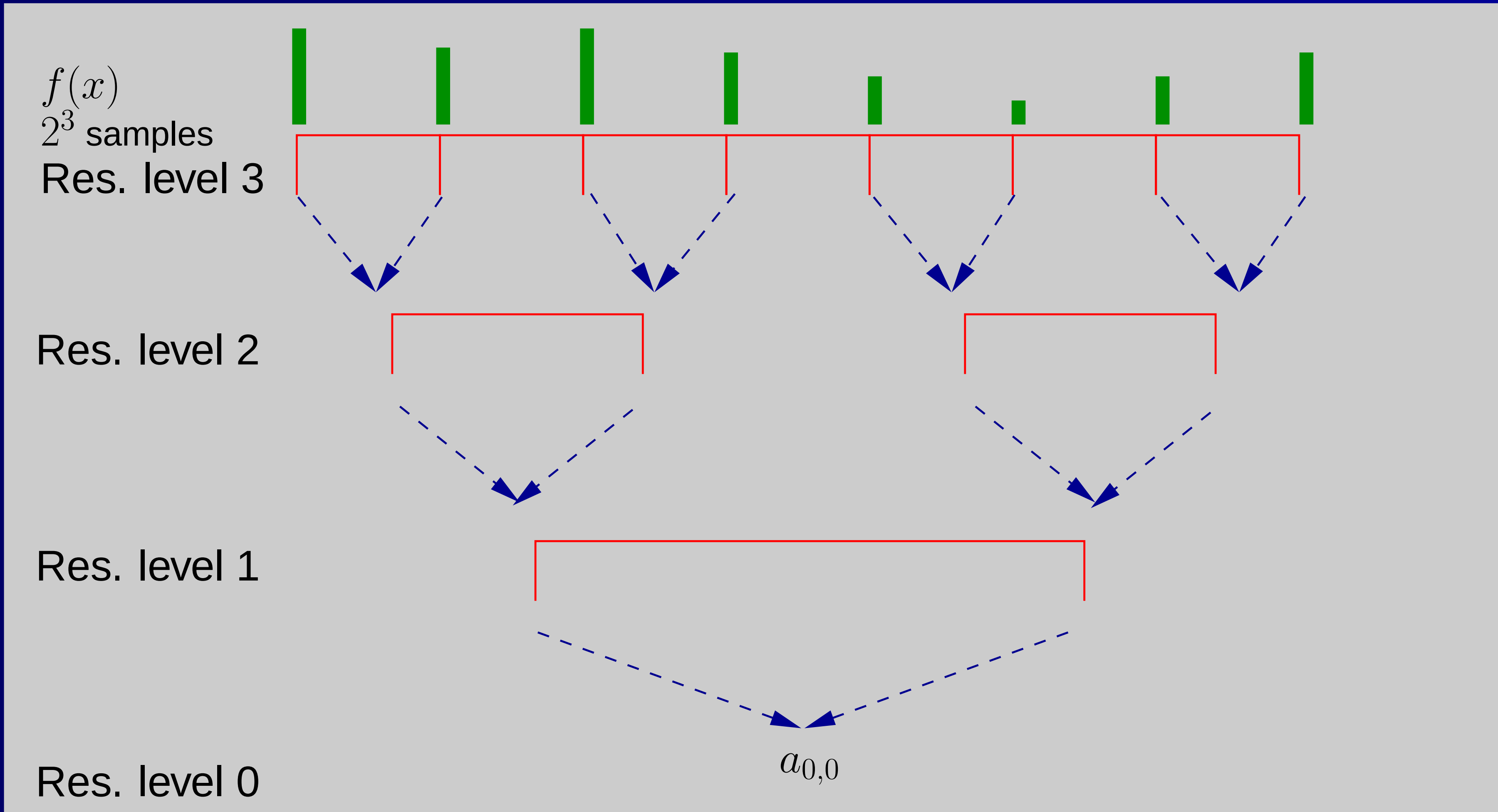
Wavelet function dilated for resolution j and interval i:

$$\psi_{j,i}(x) = 2^j \left(x - i2^{-j}\right) \psi_{[0,1[} = \begin{cases} 1 & \text{if } \; i2^{-j} \leq x < \left(i + \frac{1}{2}\right) 2^{-j} \\ -1 & \text{if } \; \left(i + \frac{1}{2}\right) 2^{-j} \leq x < (i+1) 2^{-j} \\ 0 & \text{else} \end{cases}$$

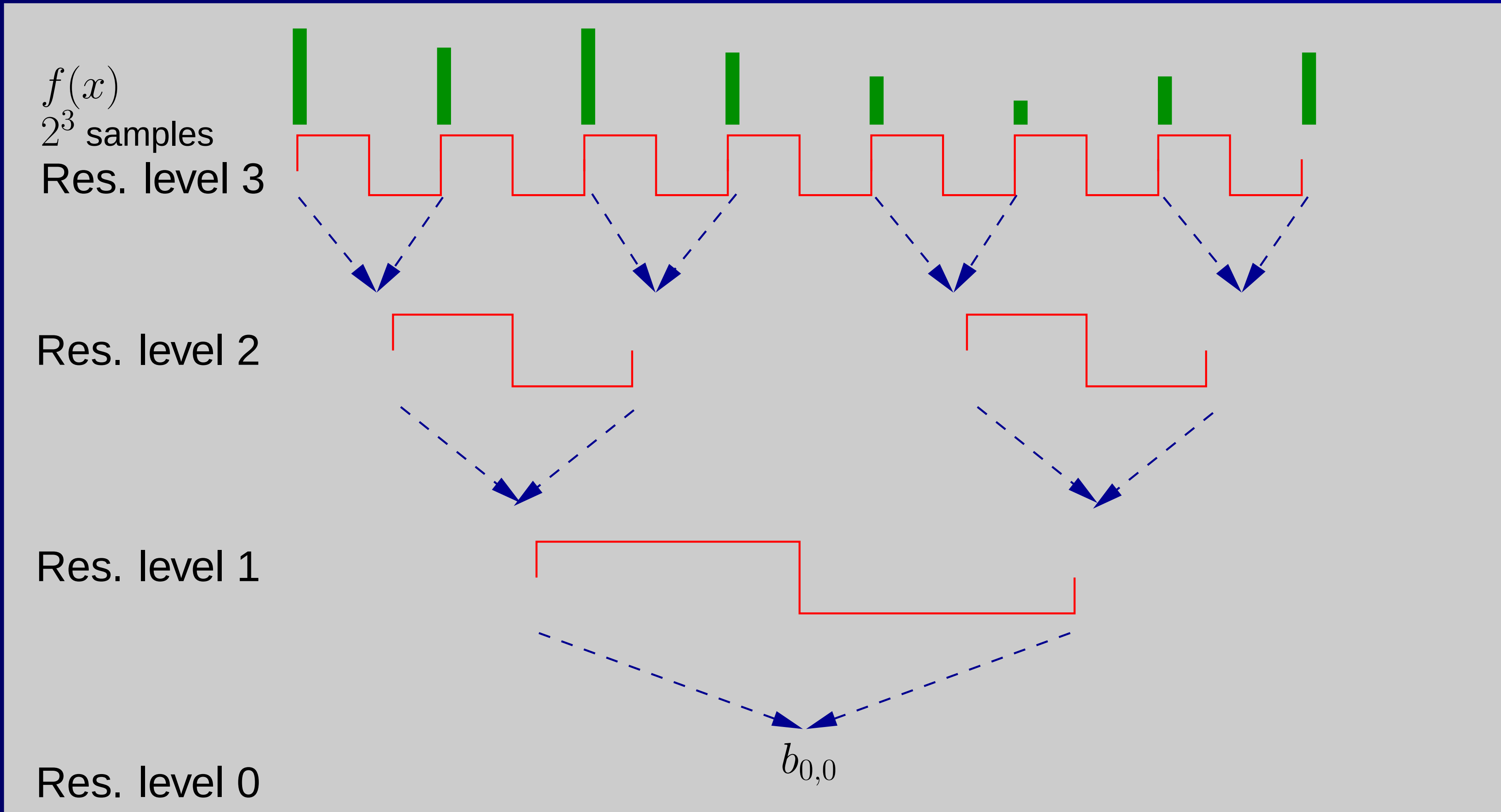


$\Rightarrow$ Difference of neighbouring samples/coeffs at res. $j+1$

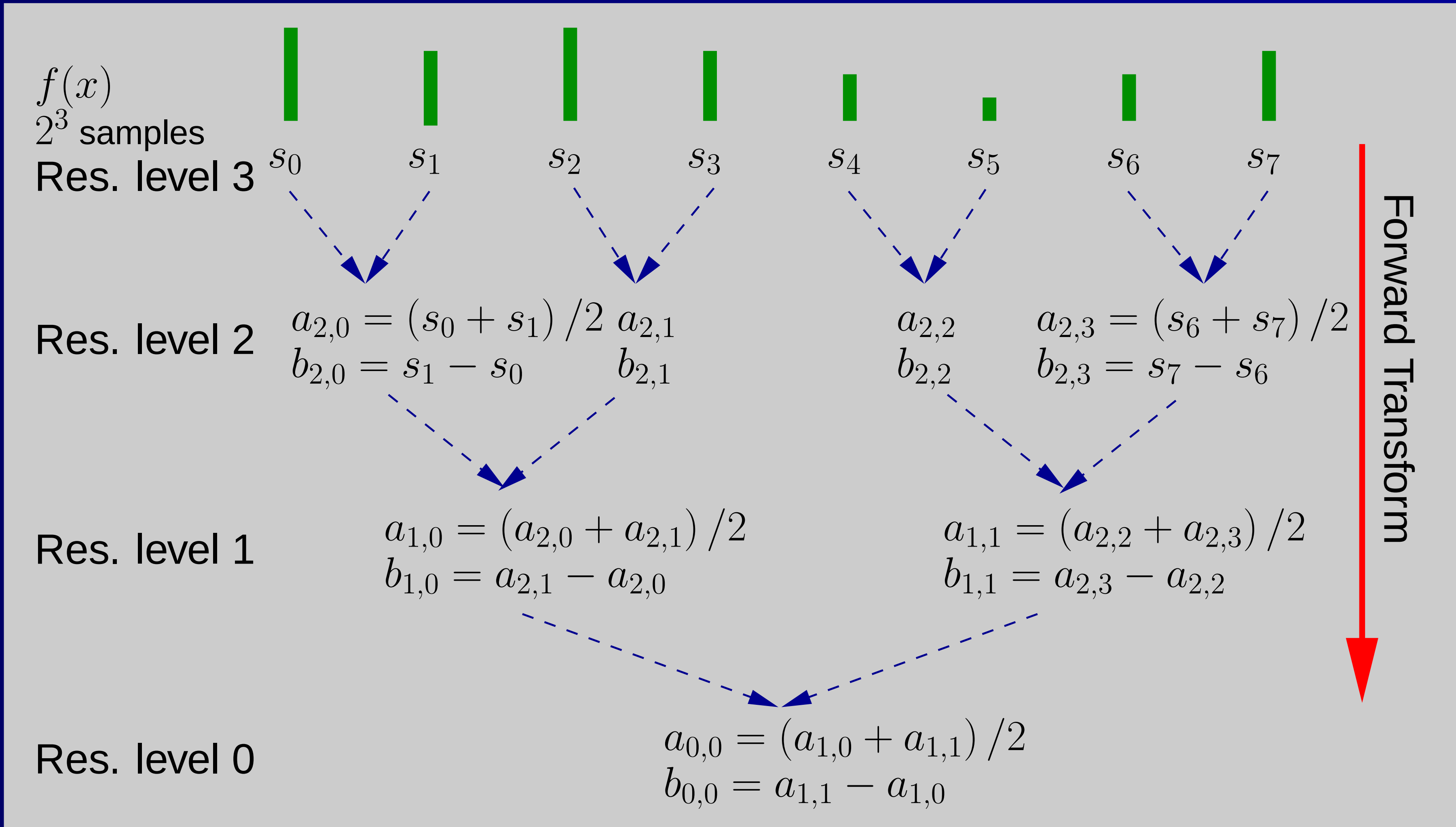
Wavelet Transform



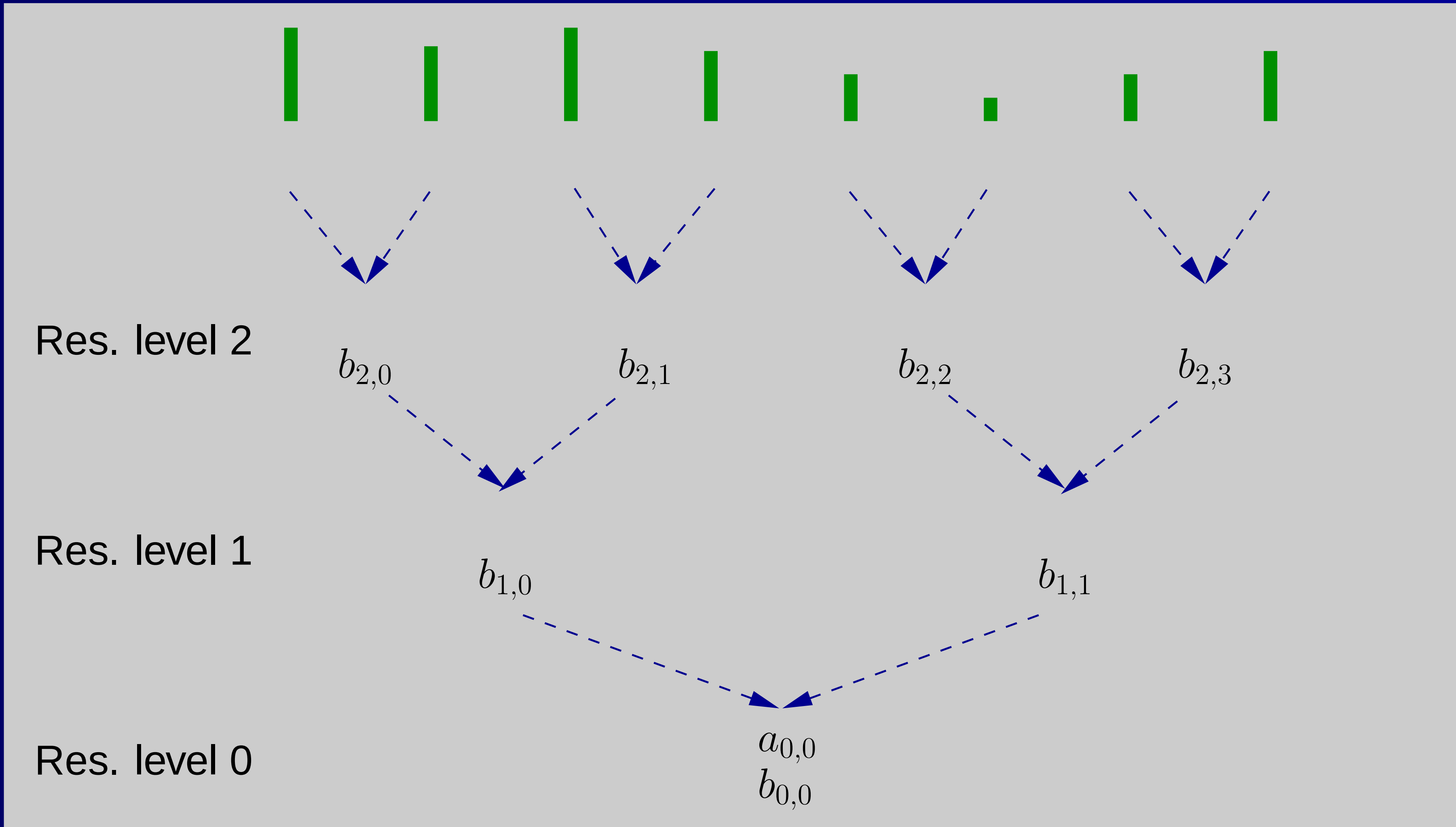
Wavelet Transform



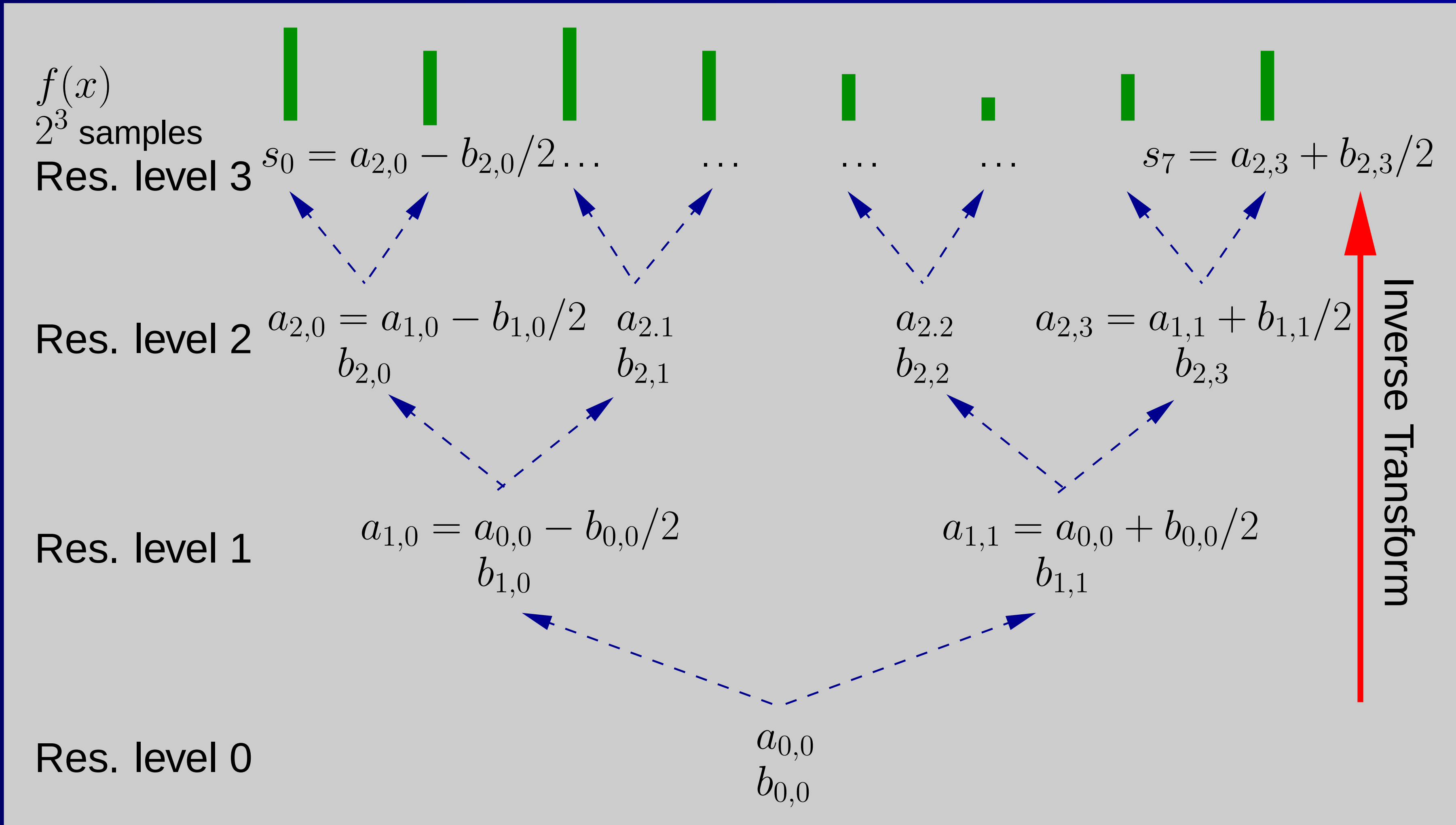
Wavelet Transform



Wavelet Transform



Wavelet Transform



Wavelet Compression

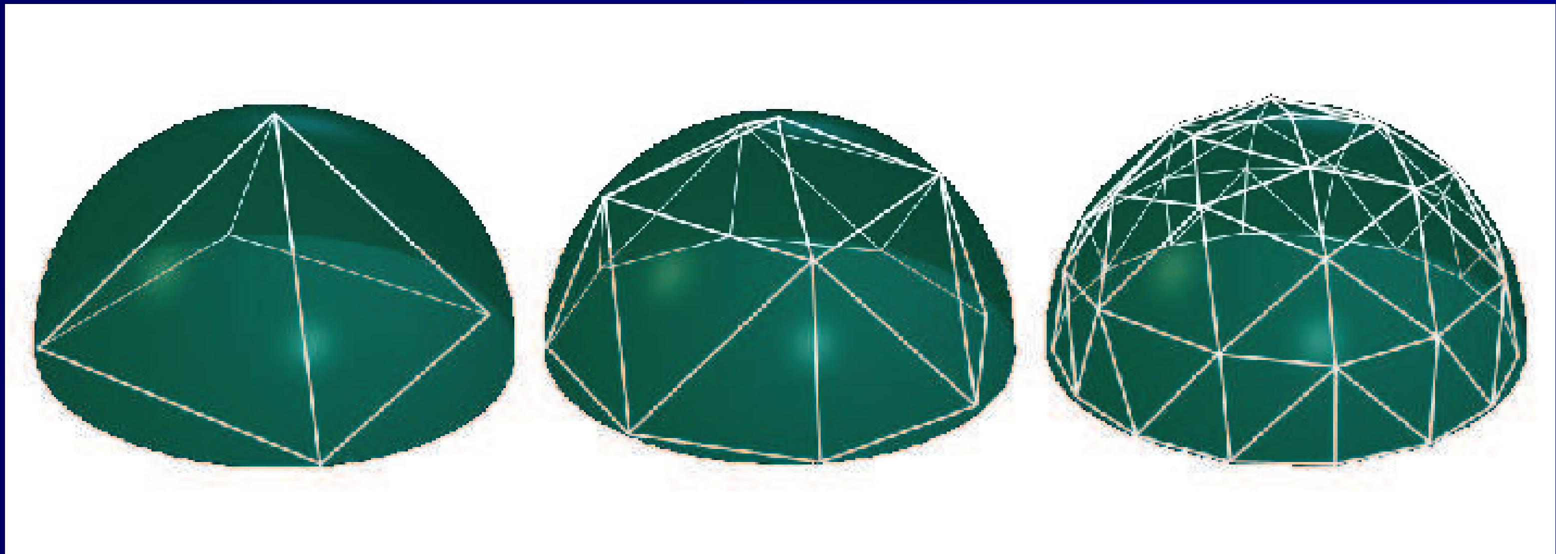
- Difference coeffs $b_{j,i} \approx 0$ where neighbouring samples at res. $j+1$ correlate
  $\Rightarrow$ remove these coeffs (minimising error) by either
  - Absolute thresholding: remove $b_{j,i}$ if $b_{j,i} \leq \tau$
  - Error bounding: remove $b_{j,0} \ldots b_{j,k-1}$ if $\sum_{i=0}^{k-1} b_{j,i} \leq \epsilon$ and $b_{j,i-1} \leq b_{j,i} < b_{j,i+1}$
- Removed coeffs implicitly set to 0 during inverse transform
- Bonus: compression induces denoising ;-)

Spherical Wavelets

- [Schröder & Sweldens, 1995]
- Adapts 2D basis functions to spherical topology
- Multiresolution analysis by recursive subdivision of spherical triangles
- Samples/coeffs live on triangle vertices
- 2D basis functions operate on triangle edges
- Represents 2D BSDF for **fixed** incident dir

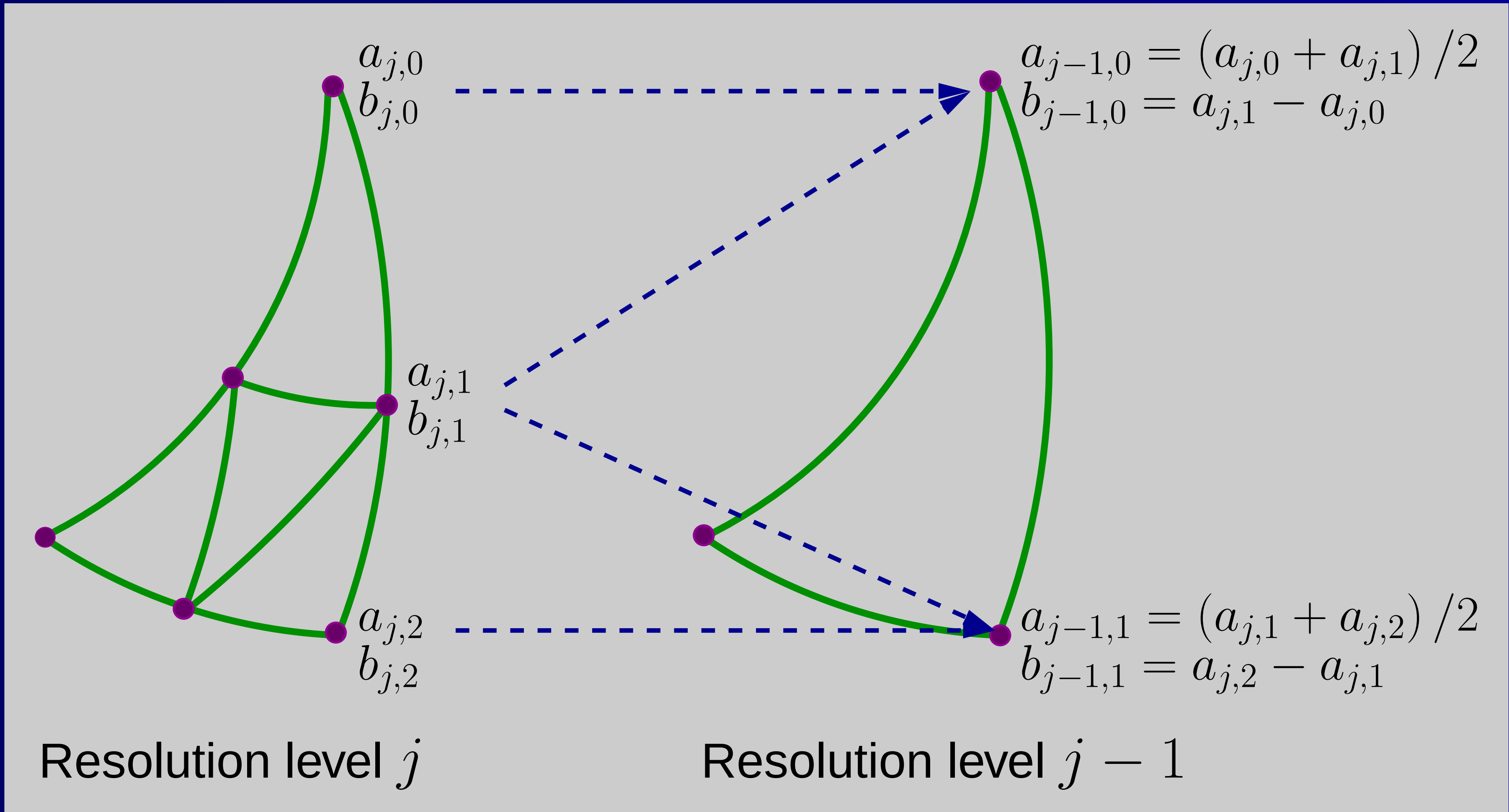
Spherical Multiresolution Analysis

Triangular subdivision at resolution levels $0 \ldots n$



Vertices at resolution level j define discrete set of m_j outgoing dirs Ω_j for fixed incident dir (ϑ_i, φ_i) with max. resolution coeffs $a_{n,k} = f_r(\vartheta_i, \varphi_i, \vartheta_{n,k}, \varphi_{n,k})$, $(\vartheta_{n,k}, \varphi_{n,k}) \in \Omega_n,\ k = 0 \ldots m_j - 1$

Spherical Wavelet Transform



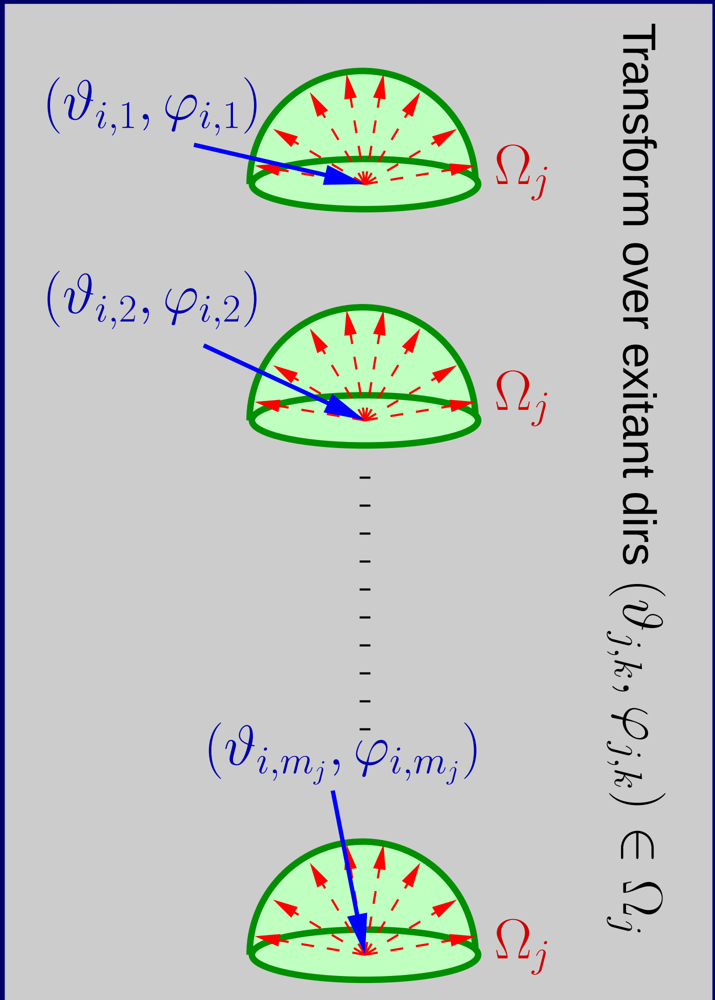
Hyperspherical Wavelets

Spherical wavelet transform only decorrelates BSDF over outgoing dirs $(\vartheta_{j,k}, \varphi_{j,k}) \in \Omega_j$ for every fixed incident dir (ϑ_i, φ_i)

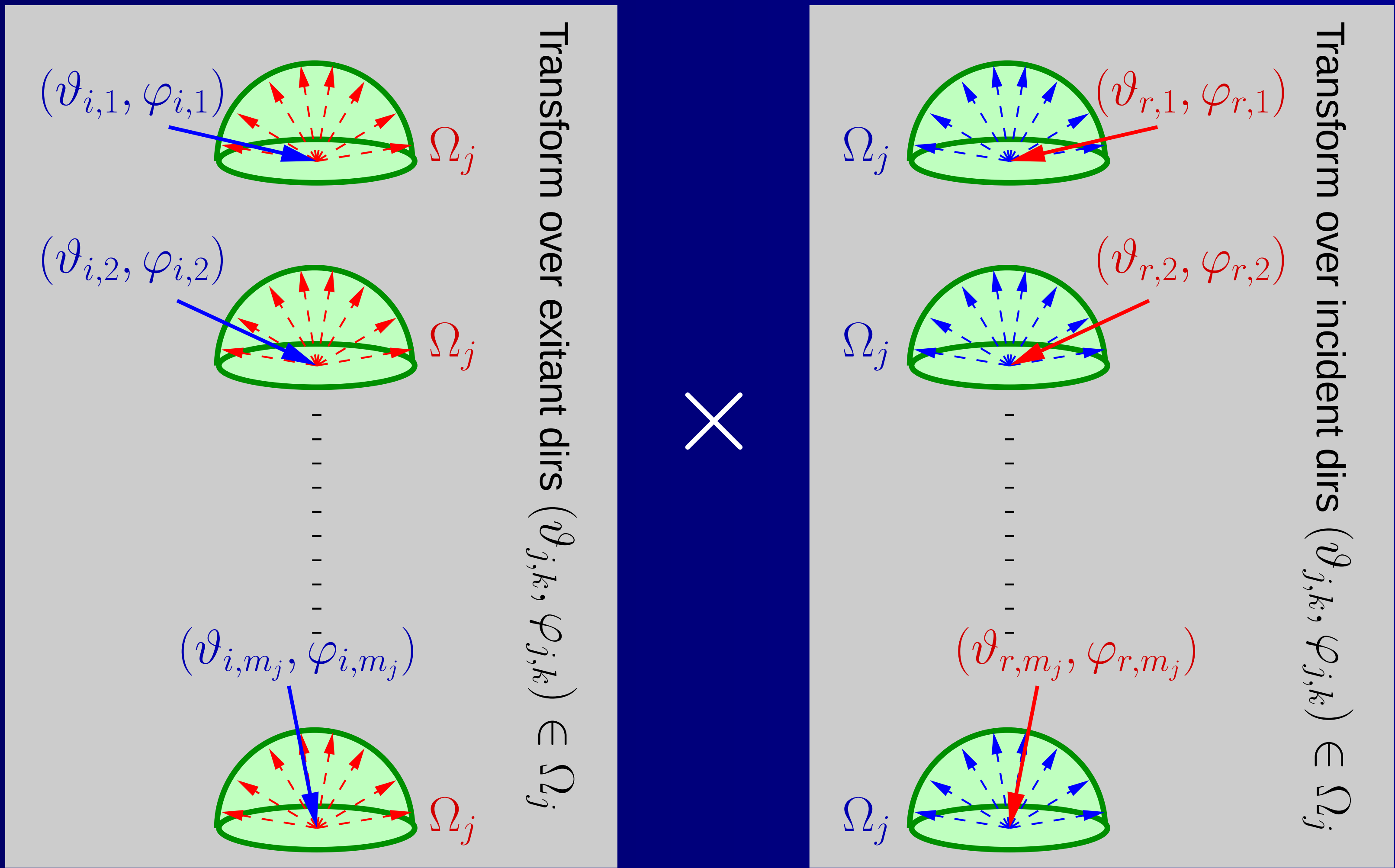
$\Rightarrow$ perform secondary wavelet transform decorrelating BSDF over incident dirs $(\vartheta_{j,k}, \varphi_{j,k}) \in \Omega_j$ for every fixed outgoing dir (ϑ_r, φ_r)

$\Rightarrow$ wavelet transform over Ω_j^2 (hyperspherical topology) by permuting in/out dirs ($= m_j^2$ coeffs)

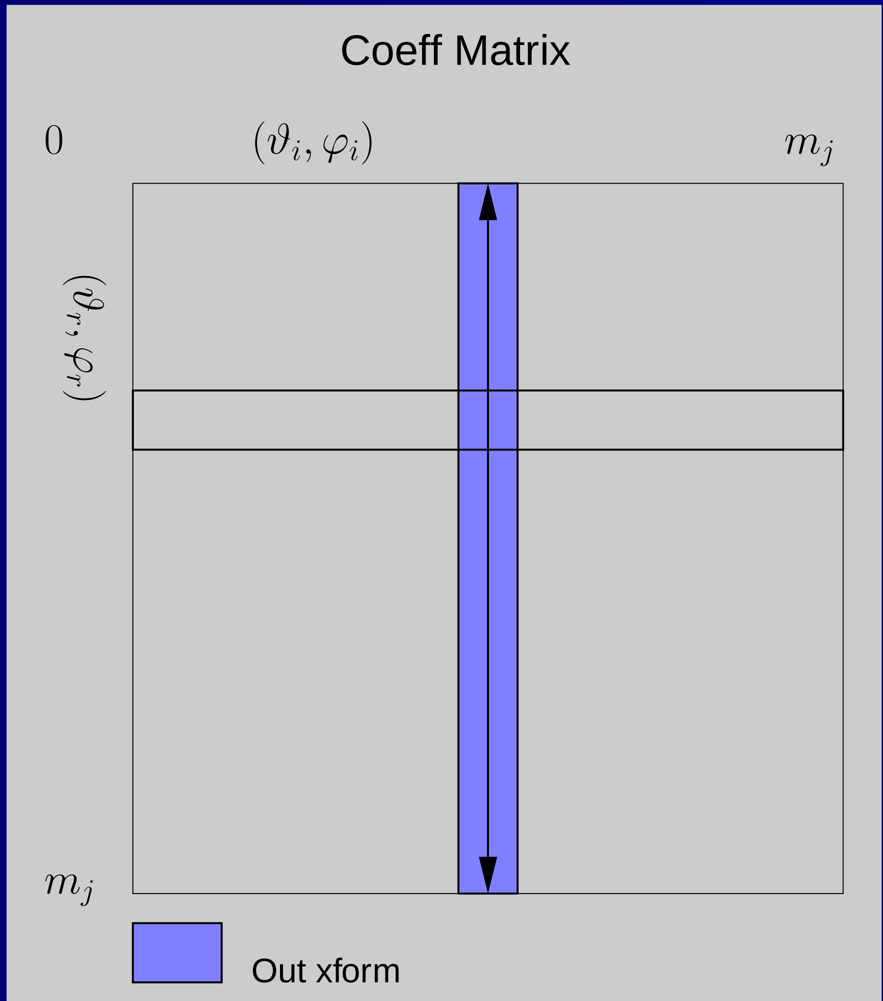
Hyperspherical Wavelets



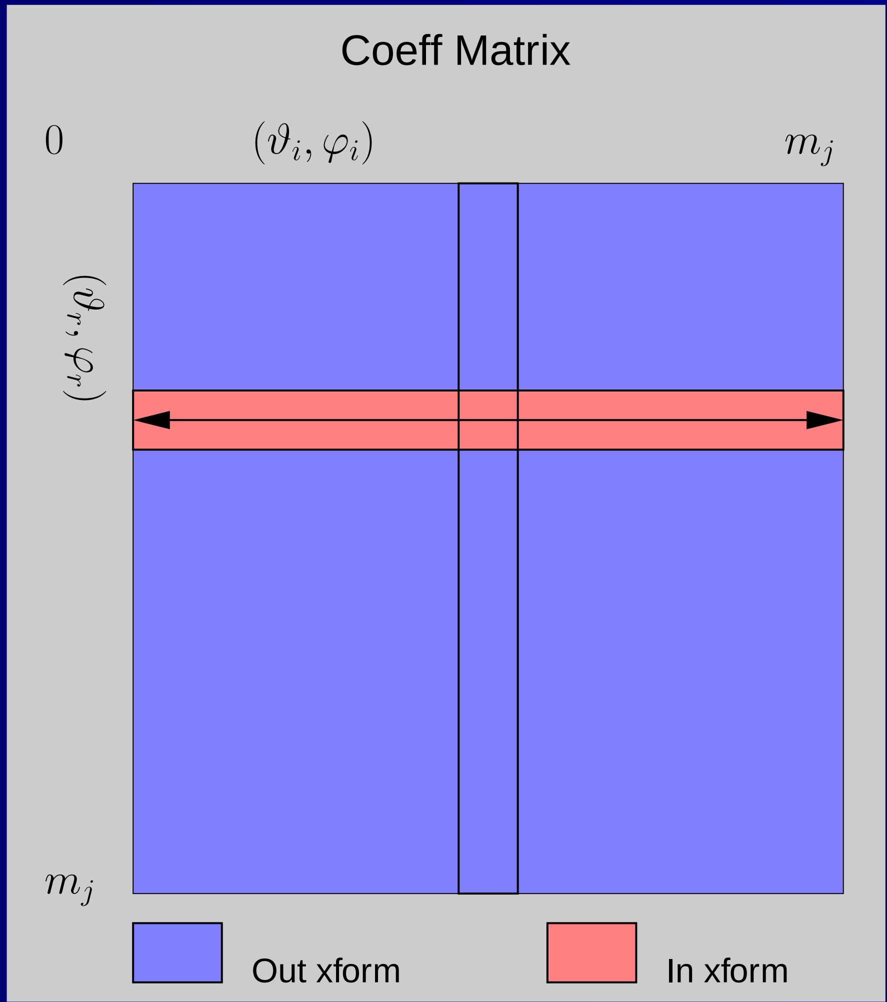
Hyperspherical Wavelets



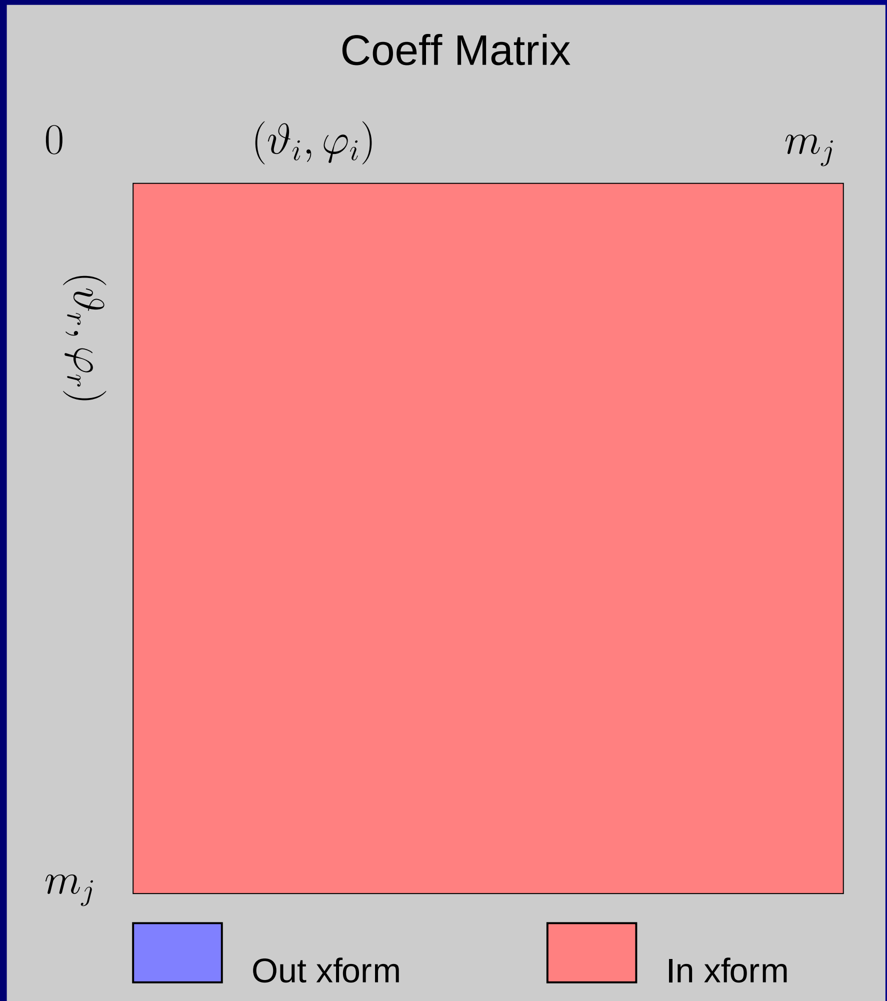
Hyperspherical Wavelets



Hyperspherical Wavelets



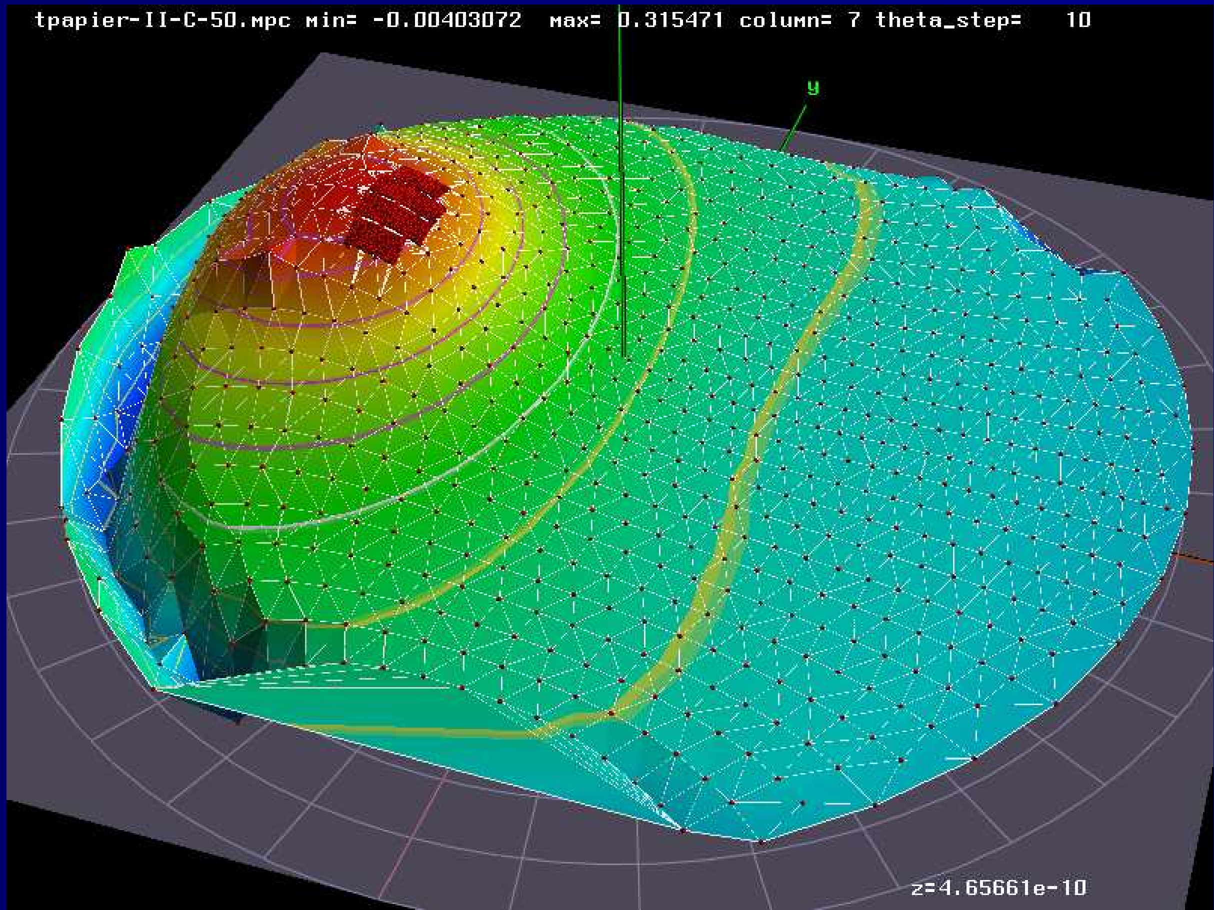
Hyperspherical Wavelets



Prototype Implementation

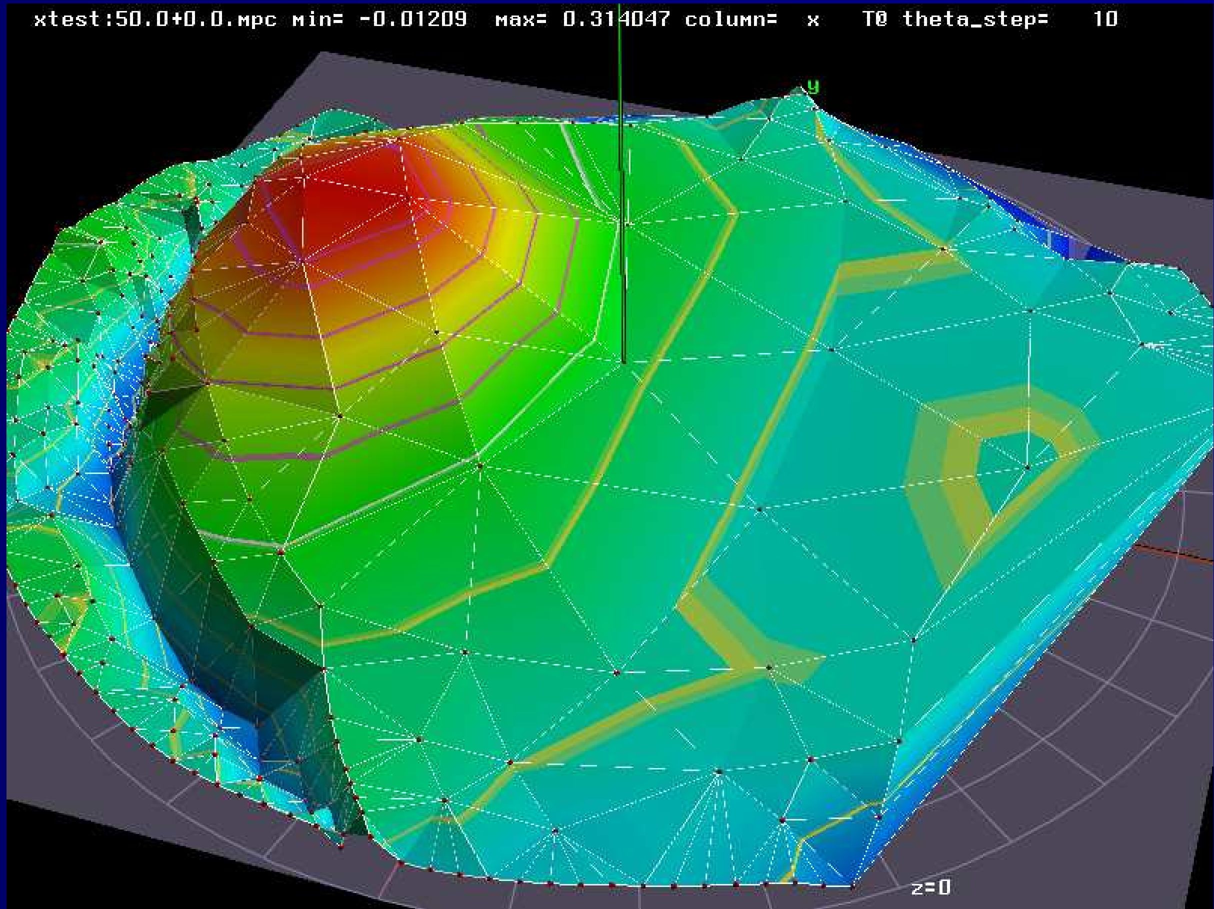
- Hideously complicated
- Uses mesh data struct to locate neighbouring triangles & shared vertices
- Measured in/out dirs associated with triangle vertices by resampling using nearest neighbour search with 4D keys $(\vartheta_i, \varphi_i, \vartheta_r, \varphi_r)$ (code borrowed from pmap)
- Handles anisotropic BSDFs, replicates isotropic BSDFs around φ

Results: Tracing Paper



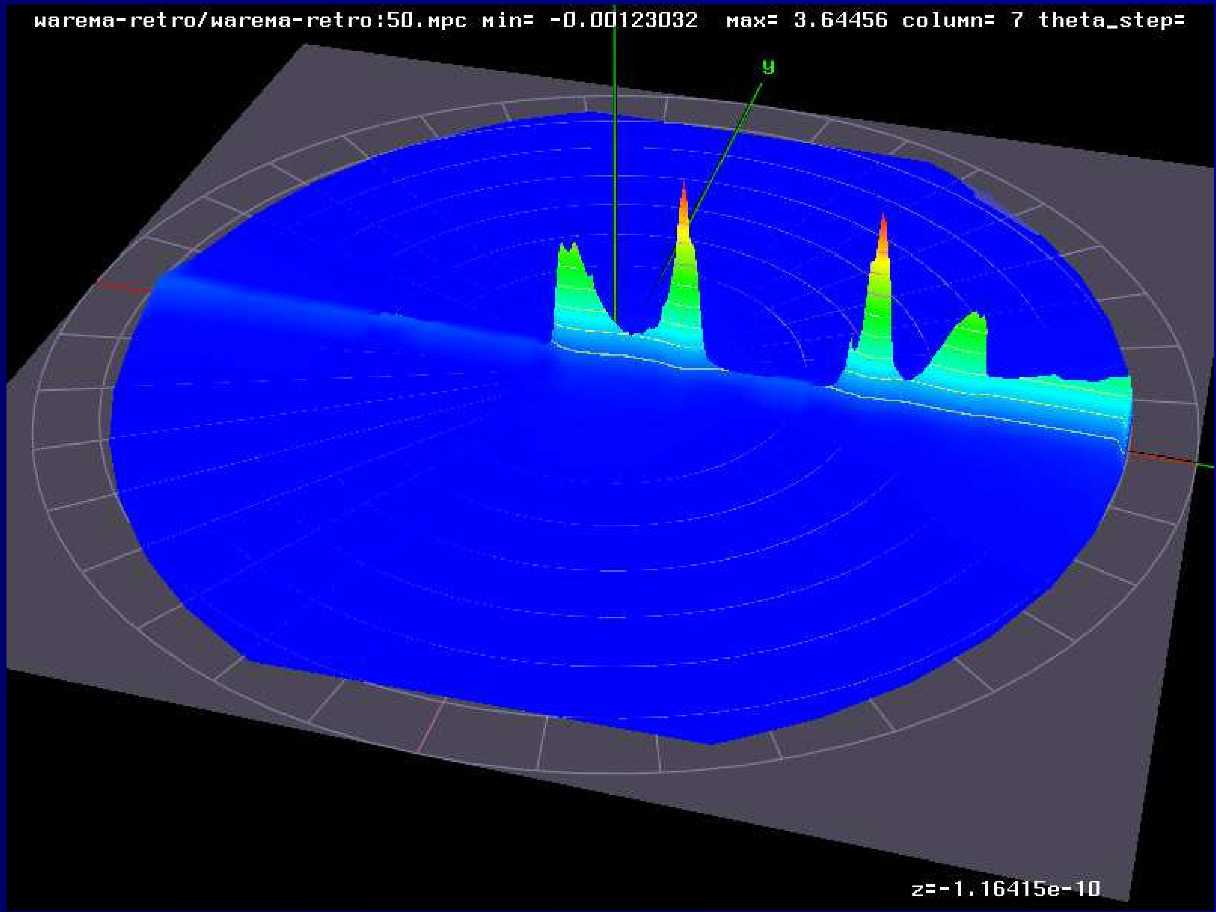
Measured BSDF

Results: Tracing Paper



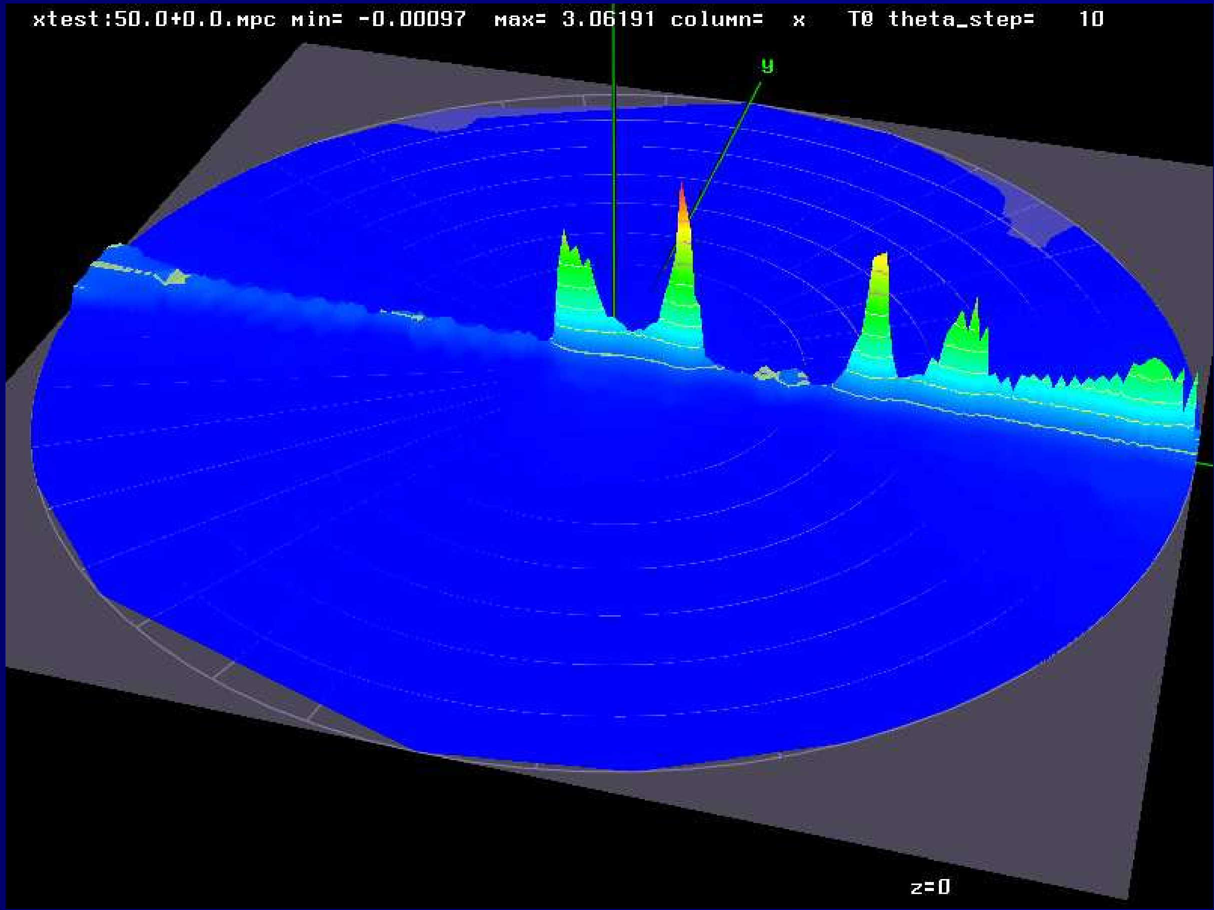
Wavelet compression, 90% coefficents removed

Results: Retroreflecting Blinds



Measured BTDF, ca. 3200 samples

Results: Retroreflecting Blinds



Wavelet compression, 88% coefficents removed

Loose Ends

- Optimised data structure (sparse arrays)?
- Integration into RADIANCE (*BSDFWavelet* material)?
- Point sampling (partial inverse transform)?
- PDF inversion for path tracing (photon map)?
- Benefit to RADIANCE community?
- Relevant to new BSDF material?

Lessons Learned

- Wavelets have great potential for BSDF applications $\Rightarrow$ successful proof of concept
- Ideal for specular BSDFs

BUT...

- Beyond original scope (photon map, validation)
- Left unpublished
- Similar concept published by Claustres, Paulin, Boucher [*BSDF Measurement Modelling Using Wavelets For Efficient Path Tracing*, 2003]