A New Ward BRDF Model with Bounded Albedo and Fitting Reflectance Data for RADIANCE

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A bidirectional reflectance distribution function (BRDF) $f$ describes the reflectance properties of a surface:

$$L_v(\theta_v, \phi_v) = \int_0^{2\pi} \int_0^{\pi/2} L_i(\theta_l, \phi_l) f(\theta_l, \phi_l; \theta_v, \phi_v) \cos \theta_l \sin \theta_l d\theta_l d\phi_l.$$ 

Physically plausible BRDFs satisfy Helmholtz reciprocity

$$f(\theta_l, \phi_l; \theta_v, \phi_v) = f(\theta_v, \phi_v; \theta_l, \phi_l)$$

and meet energy balance, i.e. have albedo $a(\theta_l, \phi_l) =$

$$\int_0^{2\pi} \int_0^{\pi/2} f(\theta_l, \phi_l; \theta_v, \phi_v) \cos \theta_v \sin \theta_v d\theta_v d\phi_v \leq 1.$$
The Ward BRDF

Ward (Computer Graphics, 1992) models anisotropic specular reflection by the BRDF

$$f_W(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi \alpha \beta} \cdot \frac{\exp \left( - \tan^2 \delta \left( \frac{\cos^2 \phi}{\alpha^2} + \frac{\sin^2 \phi}{\beta^2} \right) \right)}{4 \sqrt{\cos \theta_l \cos \theta_v}}$$

and suggests a sampling method to obtain random directions \( \vec{l} \) from the halfway vector \( \vec{h} \) given by its polar and azimuthal angles

$$\delta = \arctan \left( \sqrt{\frac{- \log(1 - s)}{\cos^2 \phi/\alpha^2 + \sin^2 \phi/\beta^2}} \right)$$

and

$$\phi = \text{atan2} \left( \beta \sin(2\pi t), \alpha \cos(2\pi t) \right),$$

where \( s \) and \( t \) are independent random numbers uniformly distributed in \([0, 1)\).
The Ward-Dür BRDF

To correct the **loss of energy** at flat angles, Dür (Journal of Graphics Tools, 2006) improved the *normalization* of the Ward BRDF to

\[
f_{\text{WD}}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi \alpha \beta} \cdot \exp \left( -\tan^2 \delta \left( \frac{\cos^2 \phi}{\alpha^2} + \frac{\sin^2 \phi}{\beta^2} \right) \right) \cdot \frac{4 \cdot \cos \theta_l \cos \theta_v}{w(\theta_l, \phi_l; \theta_v, \phi_v)}
\]

and shows that the distribution of the random direction \( \vec{I} \) generated by Ward’s sampling method has the probability density function (PDF)

\[
d_{\alpha, \beta}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{f_{\text{WD}}(\theta_l, \phi_l; \theta_v, \phi_v)}{w(\theta_l, \phi_l; \theta_v, \phi_v)}
\]

with

\[
w(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{(\cos \theta_l + \cos \theta_v)^3}{4 \cos \theta_v (1 + \cos \theta_l \cos \theta_v + \sin \theta_l \sin \theta_v \cos(\phi_v - \phi_l))}.
\]
In the mirror direction where $\theta_v = \theta_l$ and $\phi_v = \phi_l + \pi$

$$d_{\alpha,\beta}(\theta_l, \phi_l; \theta_l, \phi_l + \pi) = f_{WD}(\theta_l, \phi_l; \theta_l, \phi_l + \pi).$$

Due to the exponential decay of the Gaussian distribution

$$d_{\alpha,\beta}(\theta_l, \phi_l; \theta_v, \phi_v) \approx f_{WD}(\theta_l, \phi_l; \theta_v, \phi_v)$$

for non-grazing angles and realistic roughness values $\alpha$ and $\beta$.

For this reason the weighting factors $w(\theta_l, \phi_l; \theta_v, \phi_v)$ are usually omitted in Monte Carlo integration.
For the **direct specular component** the BRDF is evaluated in the functions `dirnorm()` and `diraniso()` in the RADIANCE source code:

\[
\int_S L_i(\vec{l}) f_{WD}(\vec{l}, \vec{v}) \, d\Omega_l \approx \sum_{m=1}^M L_i(\vec{l}(m)) f_{WD}(\vec{l}(m), \vec{v}) \, \Delta\Omega_{l(m)}
\]

The **indirect specular component** is approximated in the functions `m_normal()` and `m_aniso()` in the RADIANCE source code as

\[
\int_R L_i(\vec{l}) f_{WD}(\vec{l}, \vec{v}) \, d\Omega_l \approx L_i(\vec{l}^*),
\]

where the direction $\vec{l}^*$ is generated by Ward’s sampling method using rejection sampling.
Ward-Dür BRDF vs. PDF

Grazing angle test scene: set-up
Grazing angle test scene: luminance distributions resulting from Ward-Dür BRDF (left) and Ward's sampling method (right).
Evaluation by Ward’s sampling method

**RADIANCE 4.0**

m_normal():
324  if (!(nd.specfl & SPPURE))
325     gaussamp(r, &nd);
396  for (niter = 0; niter < MAXITER; niter++) {
397      if (niter)
398        d = frandom();
399      else
400        d = urand(ilhash(dimlist,ndims)+samplendx);

gaussamp():
417     multcolor(sr.rcol, sr.rcoef);
418     addcolor(r->rcol, sr.rcol);
419     break;
421  }
m_normal():
324  if (!(nd.specfl & SP_PURE))
...  for(i=0; i < 10000; i++)
325    gaussamp(r, &nd);

gaussamp():
396  /*  for (niter = 0; niter < MAXITER; niter++) { */
397  /*    if (niter) */
398    d = frandom();
399  /*    else */
400    d = urand(ilhash(dimlist, ndims)+samplendx);
417    multcolor(sr.rcol, sr.rcoef);
...  scalecolor(sr.rcol, (1.0/10000.0));
418    addcolor(r->rcol, sr.rcol);
419  /*      break; */
421  } */
A New Ward BRDF with Bounded Albedo

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1. Red Linoleum Floor
2. Frosted Glass (SERIS)

Conclusion
Neumann et al. (Computer Graphics Forum, 1999) criticize that the albedo of the Ward BRDF is unbounded at grazing angles, what is also valid for the Ward-Dür BRDF.

We thus propose a modification of the Ward-Dür BRDF:

\[
\begin{align*}
f_{\text{new}}(\theta_l, \phi_l; \theta_v, \phi_v) &= \frac{1}{\pi \alpha \beta} \cdot \exp \left( -\tan^2 \delta \left( \frac{\cos^2 \phi}{\alpha^2} + \frac{\sin^2 \phi}{\beta^2} \right) \right) \cdot \\
&\quad \frac{2 \left( 1 + \cos \theta_l \cos \theta_v + \sin \theta_l \sin \theta_v \cos(\phi_v - \phi_l) \right)}{(\cos \theta_l + \cos \theta_v)^4}
\end{align*}
\]
### Comparison of BRDF Models

#### New BRDF

\[
f_{\text{new}}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi \alpha \beta} \cdot \exp \left( - \tan^2 \delta \left( \frac{\cos^2 \phi}{\alpha^2} + \frac{\sin^2 \phi}{\beta^2} \right) \right) \frac{4 \langle \mathbf{l}, \mathbf{h} \rangle^2}{\langle \mathbf{h}, \mathbf{n} \rangle^4}
\]

#### Ward-Dür BRDF

\[
f_{\text{WD}}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi \alpha \beta} \cdot \exp \left( - \tan^2 \delta \left( \frac{\cos^2 \phi}{\alpha^2} + \frac{\sin^2 \phi}{\beta^2} \right) \right) \frac{4 \langle \mathbf{l}, \mathbf{n} \rangle \langle \mathbf{v}, \mathbf{n} \rangle}{\langle \mathbf{l}, \mathbf{n} \rangle \langle \mathbf{v}, \mathbf{n} \rangle}
\]

In the mirror direction where \( \theta_v = \theta_l \) and \( \phi_v = \phi_l + \pi \)

\[
f_{\text{new}}(\theta_l, \phi_l; \theta_l, \phi_l + \pi) = \frac{1}{4\pi \alpha \beta \cos^2 \theta_l} = f_{\text{WD}}(\theta_l, \phi_l; \theta_l, \phi_l + \pi)
\]
A New Ward BRDF with Bounded Albedo

The New Ward BRDF

Ward BRDF, Ward-Dür BRDF, and new BRDF at $\theta_I = 0^\circ$, $35^\circ$, and $70^\circ$ for $\alpha = \beta = 0.1$ (top) and $\alpha = \beta = 0.2$ (bottom).
A New Ward BRDF with Bounded Albedo

The New Ward BRDF

BRDF multiplied by $\cos \theta_l$ and $\cos \theta_v$ for Ward BRDF, Ward-Dür BRDF, and new BRDF at $\theta_l = 0^\circ$, $35^\circ$, and $70^\circ$ for $\alpha = \beta = 0.1$ (left) and $\alpha = \beta = 0.2$ (right).
Azimuthal variation at $\theta_l = \theta_v = 0^\circ$, $35^\circ$, and $70^\circ$ (outside to center) of BRDF multiplied by $\cos \theta_l$ and $\cos \theta_v$ for Ward BRDF, Ward-Dür BRDF, and new BRDF for $\alpha = \beta = 0.1$ (left) and $\alpha = \beta = 0.2$ (right).
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Albedo functions for $\alpha = \beta = 0.1$ for various BRDF models (left) and at grazing angles for the Ward BRDF, the Ward-Dür BRDF, and the new BRDF (right).
Numerical evidence in [DGM, AD, Computer Graphics Forum, 2010] that the albedo of the new BRDF is bounded by 1, i.e. for all $\theta_l \in [0, \pi/2)$:

$$a(\theta_l, \phi_l) = \int_0^{2\pi} \int_0^{\pi/2} f_{\text{new}}(\theta_l, \phi_l; \theta_v, \phi_v) \cos \theta_v \sin \theta_v d\theta_v d\phi_v \leq 1.$$
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Rewriting the new BRDF as

\[ f_{new}(\theta_l, \phi_l; \theta_v, \phi_v) = \]

\[
\frac{1}{\pi \alpha \beta} \cdot \exp \left( - \frac{1}{\langle \vec{l} + \vec{v}, \vec{n} \rangle^2} \cdot \left( \frac{\langle \vec{l} + \vec{v}, \vec{x} \rangle^2}{\alpha^2} + \frac{\langle \vec{l} + \vec{v}, \vec{y} \rangle^2}{\beta^2} \right) \right) \cdot \frac{\langle \vec{l} + \vec{v}, \vec{l} + \vec{v} \rangle}{\langle \vec{l} + \vec{v}, \vec{n} \rangle^4}
\]

shows that the evaluation of the BRDF for the direct specular component in the functions \texttt{dirnorm()} and \texttt{diraniso()} in the \texttt{RADIANCE} source code by

\[
\int_{S} L_l(\vec{l}) f_{new}(\vec{l}, \vec{v}) \, d\Omega_l \approx \sum_{m=1}^{M} L_l(\vec{l}(m)) f_{new}(\vec{l}(m), \vec{v}) \Delta \Omega_{l(m)}
\]

is computationally cheap.
The **indirect specular component** can be approximated in the **RADIANCE functions** `m_normal()` and `m_aniso()` by

\[
\int_{R} L_{l}(\vec{l}) f_{\text{new}}(\vec{l}, \vec{v}) \, d\Omega_{l} \approx \frac{1}{N} \sum_{n=1}^{N} L_{l}(\vec{l}^{*}_{(n)}) w_{\text{new}}(\vec{l}^{*}_{(n)}, \vec{v}),
\]

where the directions $\vec{l}^{*}_{(n)}$ are generated by Ward’s sampling method.

The **weighting factors**

\[
w_{\text{new}}(\theta_{l}, \phi_{l}; \theta_{v}, \phi_{v}) = \frac{2}{1 + \cos \theta_{v}/\cos \theta_{l}} = \frac{2}{1 + \langle \vec{v}, \vec{n} \rangle/\langle \vec{l}, \vec{n} \rangle}
\]

are **less expensive to compute** than for the Ward-Dür BRDF.

Note that $w_{\text{new}}$ is greater (less) than 1 if and only if the sampled direction is above (below) the mirror direction.
Ward-Dür BRDF vs. PDF

Grazing angle test scene: set-up
Luminance distributions laid over renderings for the Grazing Angle Test Scene: specular reflection calculated by the new BRDF (left) and by Ward’s sampling method using new weighting factors (right).
changed source code

gaussamp():
396 /* for (niter = 0; niter < MAXITER; niter++) { */
397 /* if (niter) */
398    d = frandom();
399 /* else */
400    d = urand(ilhash(dimlist, ndims) + samplendx);

417    multcolor(sr.rcol, sr.rcoef);
...    scalecolor(sr.rcol, 2.0/(1.0+r->rod/d));
...    scalecolor(sr.rcol, (1.0/10000.0));
418    addcolor(r->rcol, sr.rcol);
419 /* break; */
421 /* } */
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Conclusion
Measurement

We measured reflectance data of an isotropic red linoleum floor at Bartenbach LichtLabor in Aldrans, Austria.

Goniometer → BRDF for CIE Y

Integrating sphere → reflectance spectrum
Fitting measured BRDF data of a red linoleum

Similar to Ngan et al. (Computer Graphics Forum, 2005) we define the objective function for curve fitting as

\[ g(\theta_l, \phi_l; \theta_v, \phi_v) = \left( \text{data}(\theta_l, \phi_l; \theta_v, \phi_v) - \left( \frac{\rho \cdot \rho_d}{\pi} + \rho \cdot \rho_s \cdot f(\theta_l, \phi_l; \theta_v, \phi_v) \right) \right) \cdot \cos \theta_l, \]

where

- \( \rho = 0.175 \) is the total reflectance of the red linoleum sample,
- \( \rho_s \) is the fraction of specularly reflected light, and
- \( \rho_d = 1 - \rho_s \) is the fraction of diffusely reflected light, respectively.
Because we know $\rho = 0.175$ we can reduce the number of variables in the optimization:

$$g(\theta_l, \phi_l; \theta_v, \phi_v) =$$

$$\left( \text{data}(\theta_l, \phi_l; \theta_v, \phi_v) - 0.175 \cdot \left( \frac{1 - \rho_s}{\pi} + \rho_s \cdot f(\theta_l, \phi_l; \theta_v, \phi_v) \right) \right) \cdot \cos \theta_l$$

Then we estimate the parameters $\rho_s$ and $\alpha$ using the MATLAB routine `lsqnonlin()` such that

$$\sum_{k=1}^{222} g(\theta_l^{(k)}, \phi_l^{(k)}; \theta_v^{(k)}, \phi_v^{(k)})^2 \rightarrow \min.$$
Fitting Results

Using the provided data for

\[ \theta_l = 25^\circ, 35^\circ, \ldots, 75^\circ \quad \text{and} \quad \theta_v = 0^\circ, 2.5^\circ, 5^\circ, \ldots, 90^\circ \]

yields the following parameters and fitting errors where the optima are placed in the diagonal:

<table>
<thead>
<tr>
<th></th>
<th>(\rho_s)</th>
<th>(\alpha)</th>
<th>Ward</th>
<th>Ward-Dür</th>
<th>new BRDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_s)</td>
<td>0.08508</td>
<td>0.02605</td>
<td>0.04982</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.02935</td>
<td>0.02122</td>
<td>0.03172</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ward</td>
<td>6.8269</td>
<td>12.390</td>
<td>13.341</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ward-Dür</td>
<td>1.7e+15</td>
<td>2.8846</td>
<td>1.4e+17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>new BRDF</td>
<td>22.030</td>
<td>2.8334</td>
<td>0.9241</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The decrease of the fitting errors by factor of 7 and 3 demonstrates that, for the red linoleum, the new BRDF is better suited to approximate the measured data.
Fitting Results without $\theta_v = 90^\circ$

Ignoring the data for $\theta_v = 90^\circ$

$$\theta_l = 25^\circ, 35^\circ, \ldots, 75^\circ \quad \text{and} \quad \theta_v = 0^\circ, 2.5^\circ, 5^\circ, \ldots, 87.5^\circ$$

in the fitting procedure yields

<table>
<thead>
<tr>
<th></th>
<th>$\rho_s$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10728</td>
<td>0.04920</td>
</tr>
<tr>
<td></td>
<td>0.03411</td>
<td>0.03150</td>
</tr>
<tr>
<td>Ward</td>
<td>6.6218</td>
<td>11.156</td>
</tr>
<tr>
<td>Ward-Dür</td>
<td>23.164</td>
<td>0.9275</td>
</tr>
<tr>
<td>new BRDF</td>
<td>22.857</td>
<td>0.9179</td>
</tr>
</tbody>
</table>

The fitting results verify that the Ward-Dür BRDF and the new BRDF almost match for non-flat angles, but differ significantly at grazing angles.
Specular reflections calculated by the Ward-Dür BRDF (left) and by the new BRDF (right)
Specular highlights calculated by the Ward-Dür BRDF (left, top) and by the new BRDF (left, bottom), and relative brightness differences with Ward-Dür BRDF being the reference (right).
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Frosted Glass sample with a polished surface measured at the Solar Energy Research Institute of Singapore (SERIS)

special BSDF developed for this material using the new BRDF

→ talks by Lars Grobe and Xiaoming Yang
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Conclusion

The new BRDF

- almost equals the Ward-Dür BRDF for non-flat angles,
- is physically plausible (energy conserving, reciprocal),
- is cheap to evaluate for the direct specular component,
- admits efficient importance sampling for the indirect specular component, and
- is better suited for fitting the measured data of red linoleum.
Thanks for your attention!

Funding by the FIT-IT Program of the BMVIT (Bundesministerium für Verkehr, Innovation und Technologie) and the FFG through Grant No. 816009 is gratefully acknowledged.
Rewriting the new BRDF with respect to Ward’s sampling PDF yields

\[
a(\vec{v}) = \int_{D} \frac{2}{1 + (1 + \tan^2 \delta)/(1 - \tan^2 \delta + 2 \tan \delta \tan \theta_v \cos(\phi - \phi_v))} \, ds \, dt,
\]

where \( D = \{(s, t) \in [0, 1)^2 | 1 - \tan^2 \delta + 2 \tan \delta \tan \theta_v \cos(\phi - \phi_v) > 0\} \) denotes the feasible domain where \( \theta_l < \pi/2 \).

In the general case, i.e. when \( \delta \) is small and at non-flat angles:

\[
a(\vec{v}) \approx \int_{[0,1)^2} \frac{2}{1 + 1} \, ds \, dt = 1.
\]
Albedo of the New BRDF at grazing angles

For grazing angles, let $\delta \neq 0$ and $\theta_v \to \pi/2$. Then

$$\frac{2}{1 + (1 + \tan^2 \delta)/(1 - \tan^2 \delta + 2 \tan \delta \tan \theta_v \cos(\phi - \phi_v))} \to 2$$

and $D \to \{(s, t) \in [0, 1)^2 | \cos(\phi - \phi_v) > 0\}$.

Because the distribution of $\phi$ is point symmetric about the origin the probability that a sample ray is not rejected

$$P\{\cos(\phi - \phi_v) > 0\} = 1/2.$$ 

and thus, combining the equations yields

$$a(\vec{v}) \to 1 \quad \text{if} \quad \theta_v \to \pi/2.$$