# A New Ward BRDF Model with Bounded Albedo and Fitting Reflectance Data for RADIANCE

David Geisler-Moroder[1] Arne Dür[1] Rico Thetmeyer[2]

[1] University of Innsbruck, Austria [2] Bartenbach LichtLabor, Austria

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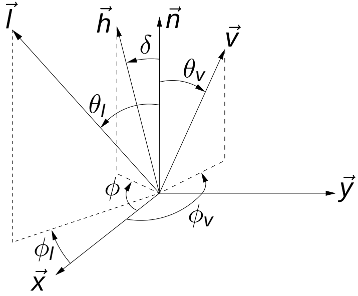
# Contents

# BRDF Basics

A bidirectional reflectance distribution function (BRDF) $f$ describes the reflectance properties of a surface:

$$L_v(\theta_v, \phi_v) = \int_0^{2\pi}\int_0^{\pi/2} L_l(\theta_l, \phi_l) f(\theta_l, \phi_l; \theta_v, \phi_v) \cos\theta_l \sin\theta_l \mathrm{d}\theta_l \mathrm{d}\phi_l .$$




## Physically plausible BRDFs

satisfy Helmholtz reciprocity

$$f(\theta_l, \phi_l; \theta_v, \phi_v) = f(\theta_v, \phi_v; \theta_l, \phi_l)$$

and meet energy balance, i.e. have albedo $a(\theta_l, \phi_l) =$

$$\int_0^{2\pi}\int_0^{\pi/2} f(\theta_l, \phi_l; \theta_v, \phi_v) \cos\theta_v \sin\theta_v \mathrm{d}\theta_v \mathrm{d}\phi_v \le 1.$$

# The Ward BRDF

Ward (Computer Graphics, 1992) models anisotropic specular reflection by the BRDF

$$f_W(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi\alpha\beta} \cdot \frac{\exp\left(-\tan^2\delta\left(\frac{\cos^2\phi}{\alpha^2} + \frac{\sin^2\phi}{\beta^2}\right)\right)}{4\sqrt{\cos\theta_l \cos\theta_v}}$$

and suggests a sampling method to obtain random directions $\vec{l}$ from the halfway vector $\vec{h}$ given by its polar and azimuthal angles

$$\delta = \arctan\left(\sqrt{\frac{-\log(1-s)}{\cos^2\phi/\alpha^2 + \sin^2\phi/\beta^2}}\right) \quad \text{and}$$

$$\phi = \text{atan2}\left(\beta\sin(2\pi t), \alpha\cos(2\pi t)\right),$$

where $s$ and $t$ are independent random numbers uniformly distributed in $[0, 1)$.

# The Ward-Dür BRDF

To correct the loss of energy at flat angles, Dür (Journal of Graphics Tools, 2006) improved the normalization of the Ward BRDF to

$$f_{\mathrm{WD}}(\theta_l,\phi_l;\theta_v,\phi_v)=\frac{1}{\pi\alpha\beta}\cdot\frac{\exp\left(-\tan^2\delta\left(\frac{\cos^2\phi}{\alpha^2}+\frac{\sin^2\phi}{\beta^2}\right)\right)}{4\cdot\cos\theta_l\cos\theta_v}$$

and shows that the distribution of the random direction $\vec{l}$ generated by Ward's sampling method has the probability density function (PDF)

$$\mathrm{d}_{\alpha,\beta}(\theta_l,\phi_l;\theta_v,\phi_v)=\frac{f_{\mathrm{WD}}(\theta_l,\phi_l;\theta_v,\phi_v)}{w(\theta_l,\phi_l;\theta_v,\phi_v)}$$

with

$$w(\theta_l,\phi_l;\theta_v,\phi_v)=\frac{(\cos\theta_l+\cos\theta_v)^3}{4\cos\theta_v\,(1+\cos\theta_l\cos\theta_v+\sin\theta_l\sin\theta_v\cos(\phi_v-\phi_l))}.$$

# Sampling the Ward-Dür BRDF

In the mirror direction where $\theta_v = \theta_l$ and $\phi_v = \phi_l + \pi$

$$\mathrm{d}_{\alpha,\beta}(\theta_l, \phi_l; \theta_l, \phi_l + \pi) = f_{\mathrm{WD}}(\theta_l, \phi_l; \theta_l, \phi_l + \pi).$$

Due to the exponential decay of the Gaussian distribution

$$\mathrm{d}_{\alpha,\beta}(\theta_l, \phi_l; \theta_v, \phi_v) \approx f_{\mathrm{WD}}(\theta_l, \phi_l; \theta_v, \phi_v)$$

for non-grazing angles and realistic roughness values $\alpha$ and $\beta$.

For this reason the weighting factors $w(\theta_l, \phi_l; \theta_v, \phi_v)$ are usually omitted in Monte Carlo integration.

# The Ward-Dür BRDF in RADIANCE

For the direct specular component the BRDF is evaluated in the functions `dirnorm()` and `diraniso()` in the RADIANCE source code:

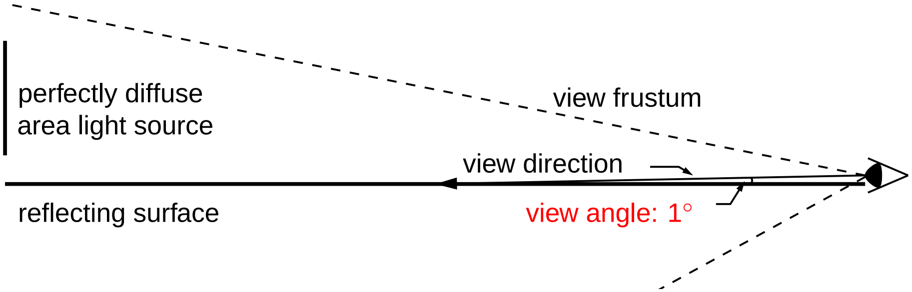
$$\int_S L_l(\vec{l})\, f_{\mathrm{WD}}(\vec{l},\vec{v})\, \mathrm{d}\Omega_l \approx \sum_{m=1}^{M} L_l(\vec{l}_{(m)})\, f_{\mathrm{WD}}(\vec{l}_{(m)},\vec{v})\, \Delta\Omega_{l_{(m)}}$$

The indirect specular component is approximated in the functions `m_normal()` and in `m_aniso()` in the RADIANCE source code as

$$\int_R L_l(\vec{l})\, f_{\mathrm{WD}}(\vec{l},\vec{v})\, \mathrm{d}\Omega_l \approx L_l(\vec{l}^{*}),$$

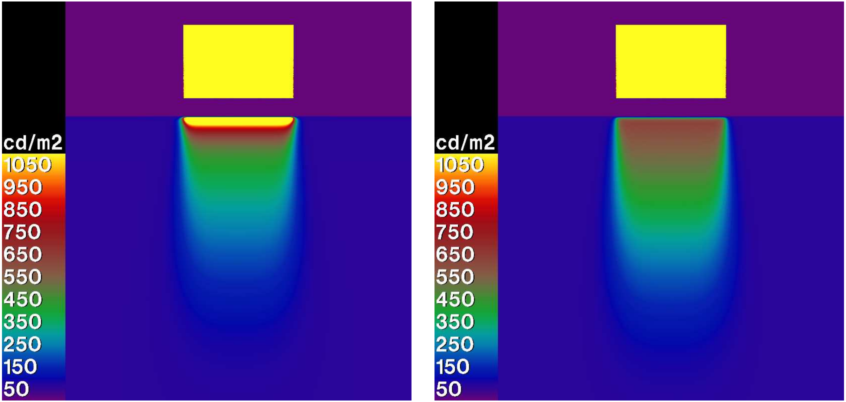
where the direction $\vec{l}^{*}$ is generated by Ward's sampling method using rejection sampling.

# Ward-Dür BRDF vs. PDF




Grazing angle test scene: set-up

# Ward-Dür BRDF vs. PDF




Grazing angle test scene: luminance distributions resulting from Ward-Dür BRDF (left) and Ward's sampling method (right).

# Evaluation by Ward's sampling method

## RADIANCE 4.0

```
m_normal():
  if (!(nd.specfl & SP_PURE))
     gaussamp(r, &nd);

  for (niter = 0; niter < MAXITER; niter++) {
     if (niter)
        d = frandom();
     else
        d = urand(ilhash(dimlist,ndims)+samplendx);

gaussamp():
            multcolor(sr.rcol, sr.rcoef);
            addcolor(r->rcol, sr.rcol);
            break;

  }
```

## changed source code

```
m_normal():
324  if (!(nd.specfl & SP_PURE))
...      for(i=0; i < 10000; i++)
325        gaussamp(r, &nd);

gaussamp():
396 /*  for (niter = 0; niter < MAXITER; niter++) { */
397 /*      if (niter) */
398         d = frandom();
399 /*      else */
400         d = urand(ilhash(dimlist,ndims)+samplendx);

417                 multcolor(sr.rcol, sr.rcoef);
...                 scalecolor(sr.rcol,(1.0/10000.0));
418                 addcolor(r->rcol, sr.rcol);
419 /*                 break; */

421 /*  } */
```

# The Proposed New Ward BRDF

Neumann et al. (Computer Graphics Forum, 1999) criticize that the albedo of the Ward BRDF is unbounded at grazing angles, what is also valid for the Ward-Dür BRDF.

**We thus propose a modification of the Ward-Dür BRDF:**

$$f_{new}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi\alpha\beta} \cdot \exp\left(-\tan^2\delta\left(\frac{\cos^2\phi}{\alpha^2} + \frac{\sin^2\phi}{\beta^2}\right)\right) \cdot \frac{2\,(1 + \cos\theta_l \cos\theta_v + \sin\theta_l \sin\theta_v \cos(\phi_v - \phi_l))}{(\cos\theta_l + \cos\theta_v)^4}$$

# Comparison of BRDF Models

## New BRDF

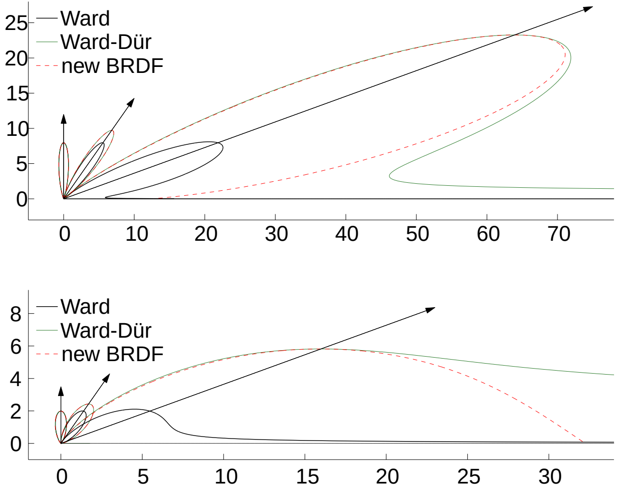
$$f_{new}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi\alpha\beta} \cdot \frac{\exp\left(-\tan^2\delta\left(\frac{\cos^2\phi}{\alpha^2} + \frac{\sin^2\phi}{\beta^2}\right)\right)}{4\langle\vec{l},\vec{h}\rangle^2\langle\vec{h},\vec{n}\rangle^4}$$

## Ward-Dür BRDF

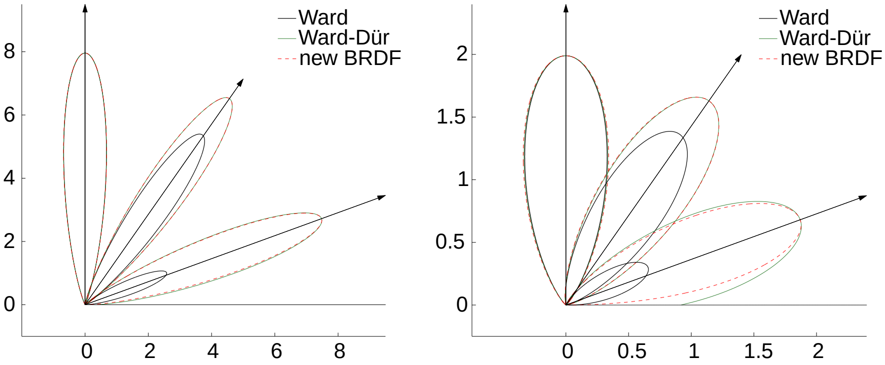
$$f_{\mathrm{WD}}(\theta_l, \phi_l; \theta_v, \phi_v) = \frac{1}{\pi\alpha\beta} \cdot \frac{\exp\left(-\tan^2\delta\left(\frac{\cos^2\phi}{\alpha^2} + \frac{\sin^2\phi}{\beta^2}\right)\right)}{4\langle\vec{l},\vec{n}\rangle\langle\vec{v},\vec{n}\rangle}$$

In the mirror direction where $\theta_v = \theta_l$ and $\phi_v = \phi_l + \pi$

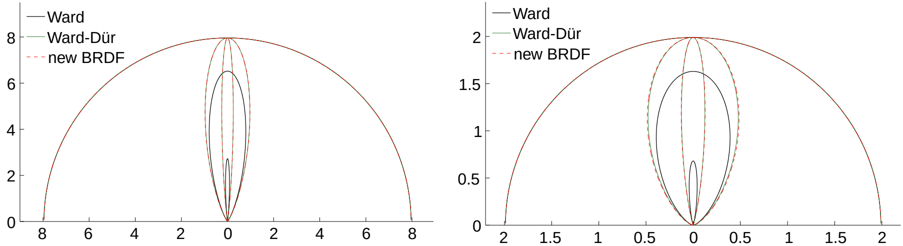
$$f_{new}(\theta_l, \phi_l; \theta_l, \phi_l + \pi) = \frac{1}{4\pi\alpha\beta\cos^2\theta_l} = f_{\mathrm{WD}}(\theta_l, \phi_l; \theta_l, \phi_l + \pi)$$




Ward BRDF, Ward-Dür BRDF, and new BRDF at $\theta_l$ = 0°, 35°, and 70° for $\alpha = \beta =$ 0.1 (top) and $\alpha = \beta =$ 0.2 (bottom).

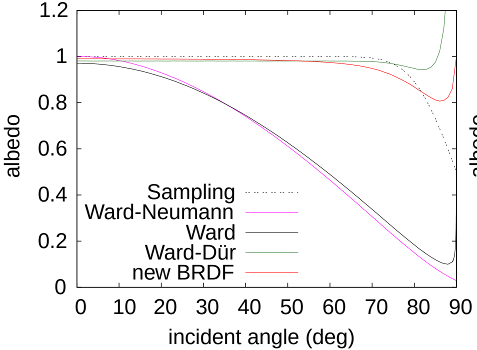


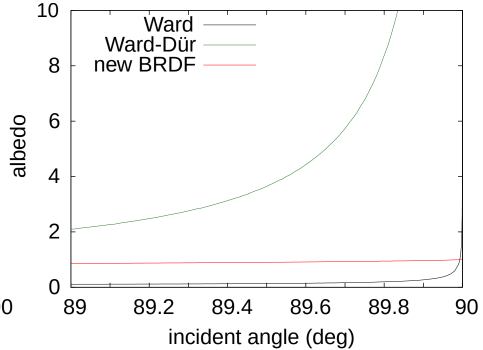

BRDF multiplied by $\cos\theta_l$ and $\cos\theta_v$ for Ward BRDF, Ward-Dür BRDF, and new BRDF at $\theta_l$ = 0°, 35°, and 70° for $\alpha = \beta =$ 0.1 (left) and $\alpha = \beta =$ 0.2 (right).




Azimuthal variation at $\theta_l = \theta_v$ = 0°, 35°, and 70° (outside to center) of BRDF multiplied by $\cos\theta_l$ and $\cos\theta_v$ for Ward BRDF, Ward-Dür BRDF, and new BRDF for $\alpha = \beta =$ 0.1 (left) and $\alpha = \beta =$ 0.2 (right).

# Albedo functions of various BRDF models



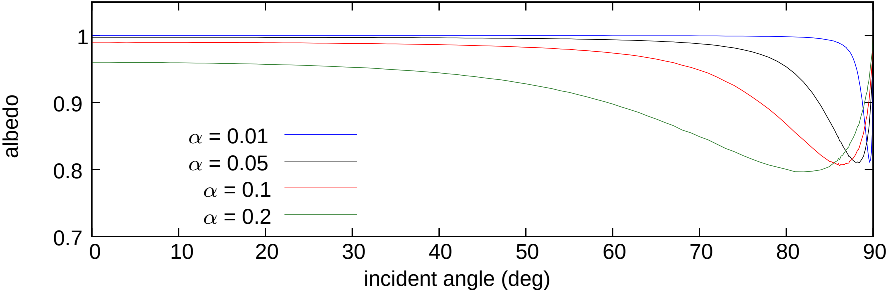

Albedo functions for $\alpha = \beta = 0.1$ for various BRDF models (left) and at grazing angles for the Ward BRDF, the Ward-Dür BRDF, and the new BRDF (right).

# Boundedness of the Albedo of the New BRDF

Numerical evidence in [DGM, AD, Computer Graphics Forum, 2010] that the albedo of the new BRDF is bounded by 1, i.e. for all $\theta_l \in [0, \pi/2)$ :

math

$$a(\theta_l, \phi_l) = \int_0^{2\pi}\int_0^{\pi/2} f_{new}(\theta_l, \phi_l; \theta_v, \phi_v) \cos\theta_v \sin\theta_v d\theta_v d\phi_v \leq 1.$$




Albedo functions of the new BRDF for varying values of $\alpha = \beta$.

# Evaluation of the New BRDF

Rewriting the new BRDF as

$$f_{new}(\theta_l, \phi_l; \theta_v, \phi_v) =$$

$$\frac{1}{\pi\alpha\beta} \cdot \exp\left( -\frac{1}{\langle \vec{l}+\vec{v}, \vec{n}\rangle^2} \cdot \left( \frac{\langle \vec{l}+\vec{v}, \vec{x}\rangle^2}{\alpha^2} + \frac{\langle \vec{l}+\vec{v}, \vec{y}\rangle^2}{\beta^2} \right) \right) \cdot \frac{\langle \vec{l}+\vec{v}, \vec{l}+\vec{v}\rangle}{\langle \vec{l}+\vec{v}, \vec{n}\rangle^4}$$

shows that the evaluation of the BRDF for the direct specular component in the functions `dirnorm()` and `diraniso()` in the RADIANCE source code by

$$\int_S L_l(\vec{l})\, f_{new}(\vec{l}, \vec{v})\, \mathrm{d}\Omega_l \approx \sum_{m=1}^{M} L_l(\vec{l}_{(m)})\, f_{new}(\vec{l}_{(m)}, \vec{v})\, \Delta\Omega_{l_{(m)}}$$

is computationally cheap.

# Importance Sampling

The indirect specular component can be approximated in the RADIANCE functions `m_normal()` and `m_aniso()` by

$$\int_R L_l(\vec{l})\, f_{new}(\vec{l},\vec{v})\, \mathrm{d}\Omega_l \approx \frac{1}{N}\sum_{n=1}^{N} L_l(\vec{l}^*_{(n)})\, w_{new}(\vec{l}^*_{(n)},\vec{v}),$$

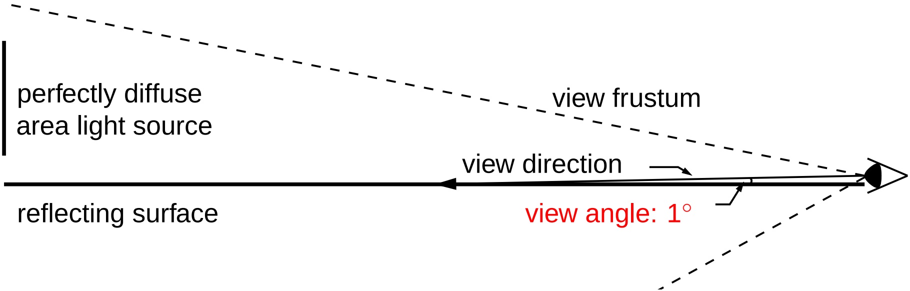
where the directions $\vec{l}^*_{(n)}$ are generated by Ward's sampling method. The weighting factors

$$w_{new}(\theta_l,\phi_l;\theta_v,\phi_v) = \frac{2}{1+\cos\theta_v/\cos\theta_l} = \frac{2}{1+\langle\vec{v},\vec{n}\rangle/\langle\vec{l},\vec{n}\rangle}$$

are less expensive to compute than for the Ward-Dür BRDF.

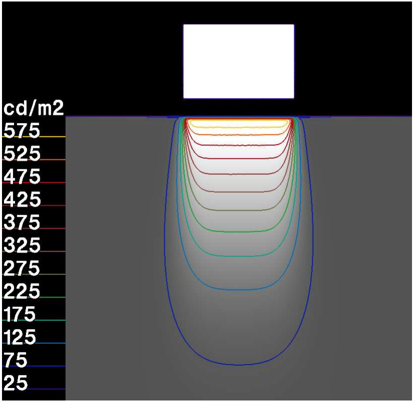
Note that $w_{new}$ is greater (less) than 1 if and only if the sampled direction is above (below) the mirror direction.

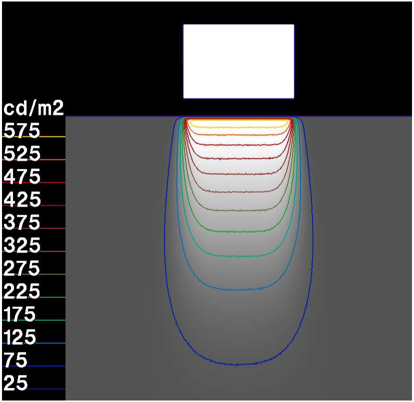
# Ward-Dür BRDF vs. PDF




Grazing angle test scene: set-up

# New BRDF vs. weighted PDF




Luminance distributions laid over renderings for the Grazing Angle Test Scene: specular reflection calculated by the new BRDF (left) and by Ward's sampling method using new weighting factors (right).

## changed source code

```
gaussamp():
/*  for (niter = 0; niter < MAXITER; niter++) { */
/*      if (niter) */
        d = frandom();
/*      else */
        d = urand(ilhash(dimlist,ndims)+samplendx);

                 multcolor(sr.rcol, sr.rcoef);
                 scalecolor(sr.rcol, 2.0/(1.0+r->rod/d));
                 scalecolor(sr.rcol,(1.0/10000.0));
                 addcolor(r->rcol, sr.rcol);
/*                   break; */

/*  } */
```

# Measurement

We measured reflectance data of an isotropic red linoleum floor at Bartenbach LichtLabor in Aldrans, Austria.

Goniometer
→ BRDF for CIE Y


Integrating sphere
→ reflectance spectrum

# Fitting measured BRDF data of a red linoleum

Similar to Ngan et al. (Computer Graphics Forum, 2005) we define the objective function for curve fitting as

$$g(\theta_l, \phi_l; \theta_v, \phi_v) =$$

$$\left(\mathrm{data}(\theta_l, \phi_l; \theta_v, \phi_v) - \left(\frac{\rho \cdot \rho_d}{\pi} + \rho \cdot \rho_s \cdot f(\theta_l, \phi_l; \theta_v, \phi_v)\right)\right) \cdot \cos\theta_l,$$

where

- $\rho = 0.175$ is the total reflectance of the red linoleum sample,
- $\rho_s$ is the fraction of specularly reflected light, and
- $\rho_d = 1 - \rho_s$ is the fraction of diffusely reflected light, respectively.

# Fitting measured BRDF data of a red linoleum

Because we know $\rho = 0.175$ we can reduce the number of variables in the optimization:

$$g(\theta_l, \phi_l; \theta_v, \phi_v) =$$

$$\left(\mathrm{data}(\theta_l, \phi_l; \theta_v, \phi_v) - 0.175 \cdot \left(\frac{1-\rho_s}{\pi} + \rho_s \cdot f(\theta_l, \phi_l; \theta_v, \phi_v)\right)\right) \cdot \cos\theta_l$$

Then we estimate the parameters $\rho_s$ and $\alpha$ using the MATLAB routine *lsqnonlin()* such that

$$\sum_{k=1}^{222} g(\theta_l^{(k)}, \phi_l^{(k)}; \theta_v^{(k)}, \phi_v^{(k)})^2 \to \min .$$

# Fitting Results

Using the provided data for

$$\theta_l = 25°, 35°, \ldots, 75° \quad \text{and} \quad \theta_v = 0°, 2.5°, 5°, \ldots, 90°$$

yields the following parameters and fitting errors where the optima are placed in the diagonal:

| | | | |
|---|---|---|---|
| $\rho_s$ | 0.08508 | 0.02605 | 0.04982 |
| $\alpha$ | 0.02935 | 0.02122 | 0.03172 |
| Ward | 6.8269 | 12.390 | 13.341 |
| Ward-Dür | 1.7e+15 | 2.8846 | 1.4e+17 |
| new BRDF | 22.030 | 2.8334 | 0.9241 |

The decrease of the fitting errors by factor of 7 and 3 demonstrates that, for the red linoleum, the new BRDF is better suited to approximate the measured data.

# Fitting Results without $\theta_v = 90^\circ$

Ignoring the data for $\theta_v = 90^\circ$

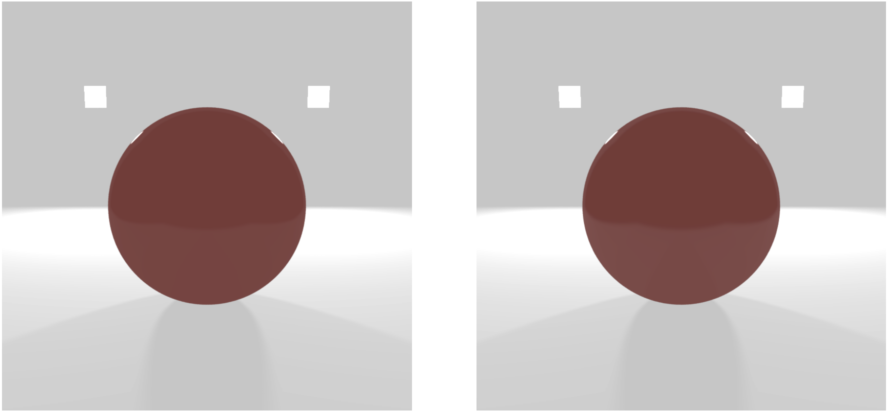
$$\theta_l = 25^\circ, 35^\circ, \ldots, 75^\circ \quad \text{and} \quad \theta_v = 0^\circ, 2.5^\circ, 5^\circ, \ldots, 87.5^\circ$$

in the fitting procedure yields

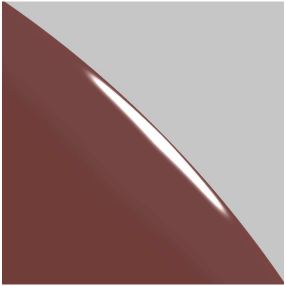
| | | | |
|---|---|---|---|
| $\rho_s$ | 0.10728 | 0.04920 | 0.04981 |
| $\alpha$ | 0.03411 | 0.03150 | 0.03171 |
| Ward | 6.6218 | 11.156 | 11.129 |
| Ward-Dür | 23.164 | 0.9275 | 0.9281 |
| new BRDF | 22.857 | 0.9179 | 0.9173 |

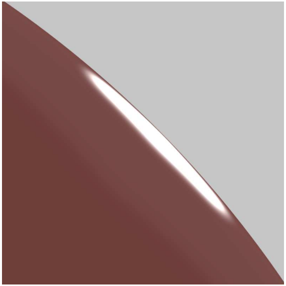
The fitting results verify that the Ward-Dür BRDF and the new BRDF almost match for non-flat angles, but differ significantly at grazing angles.

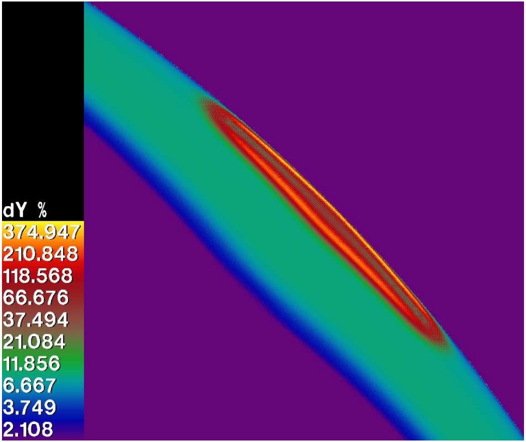
# Rendering with fitted parameters



Specular reflections calculated by the Ward-Dür BRDF (left) and by the new BRDF (right)








Specular highlights calculated by the Ward-Dür BRDF (left, top) and by the new BRDF (left, bottom), and relative brightness differences with Ward-Dür BRDF being the reference (right).

# Frosted Glass (SERIS)

- frosted glass sample with a polished surface measured at the Solar Energy Research Institute of Singapore (SERIS)
- special BSDF developed for this material using the new BRDF
- → talks by Lars Grobe and Xiaoming Yang

1. A New Ward BRDF with Bounded Albedo
   - Introduction and Related Work
   - The New Ward BRDF
   - Albedo of the New BRDF
   - Implementation

2. Fitting Measured BRDF/BSDF Data
   - Red Linoleum Floor
   - Frosted Glass (SERIS)

3. Conclusion

## Conclusion

The new BRDF

- almost equals the Ward-Dür BRDF for non-flat angles,
- is physically plausible (energy conserving, reciprocal),
- is cheap to evaluate for the direct specular component,
- admits efficient importance sampling for the indirect specular component, and
- is better suited for fitting the measured data of red linoleum.

**Thanks for your attention!**

Funding by the FIT-IT Program of the BMVIT (Bundesministerium für Verkehr, Innovation und Technologie) and the FFG through Grant No. 816009 is gratefully acknowledged.

# Albedo of the New BRDF at non-flat angles

Rewriting the new BRDF with respect to Ward's sampling PDF yields

$$a(\vec{v}) = \iint_D \frac{2}{1 + (1 + \tan^2\delta)/(1 - \tan^2\delta + 2\tan\delta\tan\theta_v\cos(\phi - \phi_v))}\,\mathrm{d}s\,\mathrm{d}t,$$

where $D = \{(s,t) \in [0,1)^2 | 1 - \tan^2\delta + 2\tan\delta\tan\theta_v\cos(\phi - \phi_v) > 0\}$ denotes the feasible domain where $\theta_l < \pi/2$.

In the general case, i.e. when $\delta$ is small and at non-flat angles:

$$a(\vec{v}) \approx \iint_{[0,1)^2} \frac{2}{1+1}\mathrm{d}s\mathrm{d}t = 1.$$

# Albedo of the New BRDF at grazing angles

For grazing angles, let $\delta \neq 0$ and $\theta_v \to \pi/2$. Then

go back

$$\frac{2}{1 + (1 + \tan^2 \delta)/(1 - \tan^2 \delta + 2 \tan \delta \tan \theta_v \cos(\phi - \phi_v))} \to 2$$

$$\text{and} \quad D \to \{(s, t) \in [0, 1)^2 | \cos(\phi - \phi_v) > 0\}.$$

Because the distribution of $\phi$ is point symmetric about the origin the probability that a sample ray is not rejected

$$\mathrm{P}\{\cos(\phi - \phi_v) > 0\} = 1/2.$$

and thus, combining the equations yields

$$a(\vec{v}) \to 1 \quad \text{if} \quad \theta_v \to \pi/2.$$